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Galois theory for uni-serial rings.

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In a previous paper[1], I have given a new method to the theory of simple rings, which enables us in particular to prove the fundamental theorem of simple rings in a quite natural way as well as to extend the Jacobson's Galois theory[2] from quasi-fields to simple rings; our principal method was in fact to embed the simple ring into an absolute endomorphism ring (of a certain module) and take commuter ring in it. In this paper we shall show that by means of the similar method these results can be extended completely to the uni-serial case[3] and shall obtain some other detailed results which have significance even in the case of simple rings. Further, after establishing the Galois theory, we shall give a new and simpler proof to the existence theorem of normal bases[4].

Throughout the present paper, we mean by a ring always one possessing an unit element and by a subring always one whose unit element coincides with that of the original ring, and when we deal with a module with operator-ring we assume always that the unit element of the latter operates on the former as the identity endomorphism. Further, when $\mathfrak{S}$ is a subring of a ring $\mathfrak{R}$, we denote by $V_{\mathfrak{R}}(\mathfrak{S})$ the commuter ring of $\mathfrak{S}$ in $\mathfrak{R}$.

For the sake of completeness, let us begin with the following consideration concerning moduli with operator-ring:

§1. Moduli with operator-ring and their submoduli.

Lemma 1.[5] *Let $\mathfrak{R}$ be a two-sided simple ring*[6] *with the center* Z[7] *and*

1) Azumaya [2]. Cf. also Nakayama-Azumaya [13].

2) Jacobson [6].

3) While their extention to irreducible rings is treated in Nakayama-Azumaya [13].

4) In case of quasi-fields, this theorem was proved in Nakayama [12]. The same method can readily be transferred to the case of simple rings. However, it can no longer, as it seems to the writer, apply to our case.

5) Cf. Kurosh [8].

6) By a two-sided simple ring we understand a ring which possesses no non-trivial two-sided ideal, while if a two-sided simple ring satisfies the minimum condition for right (or equivalently left) ideals we call it a simple ring.

7) Z forms a (commutative) field.