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Notes on Fourier Analysis (XI): On the absolute summability of Fourier series.

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In the present note the author discusses three different problems concerning the absolute Cesàro summability of Fourier series. Firstly we prove a series theorem and as corollaries we get some analoga of the absolute convergence theorems of Fourier series (in §1). In §2 we prove theorems concerning absolute summability factors. Finally, in §3, we prove a localization theorem of the absolute summability and show that the analogue of the Denjoy-Lusin theorem does not hold in general.

§1. **Theorem 1.** *If* $\sum_{n=1}^{\infty}|u_n|$, *then* $\sum_{n=1}^{\infty} u_n/A_n^{(\gamma)}$ $(0<\gamma<1)$ *is* $|C,-\gamma|$-*summable, where* $A_n^{(\gamma)}=\binom{n+\gamma}{n}$.

Proof. By $s_n^{(\delta)}$ we denote the n-th Cesàro mean of order δ (>-1) of the series $\sum_{n=1}^{\infty} x_n$. Then[1)]

$$x_n^{(-\gamma)}\equiv s_n^{(-\gamma)}-s_{n-1}^{(-\gamma)}=\frac{1}{nA_n^{(-\gamma)}}\sum_{k=1}^{n} kA_{n-k}^{(-\gamma-1)}x_k.$$

Putting $x_n=u_n/A_n^{(\gamma)}$, we have

$$|x_n^{(-\gamma)}|\leqq\frac{-1}{nA_n^{(-\gamma)}}\sum_{k=1}^{n-1}A_{n-k}^{(-\gamma-1)}k\frac{|u_k|}{A_n^{(\gamma)}}+|u_n|,$$

$$\sum_{n=2}^{N+1}|x_n^{(-\gamma)}|\leqq\sum_{p=1}^{N}\frac{-1}{(p+1)A_{p+1}^{(-\gamma)}}\sum_{k=1}^{p}A_{p-k+1}^{(-\gamma-1)}\frac{k\,|u_k|}{A_k^{(\gamma)}}+\sum_{n=2}^{N}|u_n|$$

$$\leqq\sum_{k=1}^{N}\frac{k\,|u_k|}{A_k^{(\gamma)}}\sum_{p=k}^{N}\frac{-A_{p-k+1}^{(-\gamma-1)}}{(p+1)A_{p+1}^{(-\gamma)}}+\sum_{n=2}^{N}|u_n|,$$

where

$$\sum_{p=k}^{N}\frac{-A_{p-k+1}^{(-\gamma-1)}}{(p+1)A_{p+1}^{(-\gamma)}}=\sum_{i=1}^{N-k+1}\frac{-A_i^{(-\gamma-1)}}{(k+i)A_{k+i}^{(-\gamma)}}$$

$$\leqq\frac{1}{k^{1-\gamma}}\sum_{i=1}^{N-k+1}(-A_i^{(-\gamma-1)})\leqq\frac{1}{k^{1-\gamma}}\sum_{i=1}^{\infty}(-A_i^{(-\gamma-1)}).$$

1) This formula is due to Kogbetlianz [6].