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# Notes on Fourier Analysis (X).
# On the summability of Fourier series.

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**1.** Let $\phi(x)$ be an L-integrable and periodic function with period $2\pi$. For any $k \geqq 0$ and any $\alpha \geqq 0$ we define $\Psi_k(x)$ and $\Psi_k^\alpha(x)$ the formula:

$$\Psi_k(x) = \frac{1}{\Gamma(k)}\int_x^\pi \left(\log\frac{u}{x}\right)^{k-1}\phi(u)\frac{du}{u},$$
$$\Psi_0(x) = \phi(x)$$

and

$$\Psi_k^\alpha(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-v)^{\alpha-1}\Psi_k(v)\,dv,$$
$$\Psi_k^0(x) = \Psi_k(x).$$

If $\Psi_k^\alpha(x)/x^\alpha(\log 1/x)^k = o(1)$ as $x\to 0$, we say that $\phi(x)$ is $(\alpha, k)$-continuous at $x=0$.

Let $\sum a_n$ be a given series and $A(u) \equiv \sum_{v<u} a_v$ be its partial sum. For any $k \geqq 0$ we define Riesz's sum $R_k(\omega)$ of order $k$ by

$$R_k(\omega) = \sum_{n<\omega}\left(\log\frac{\omega}{n}\right)^k a_n = \frac{1}{\Gamma(k)}\int_1^\omega \left(\log\frac{\omega}{u}\right)^{k-1}\frac{A(u)}{n}du,$$
$$R_0(\omega) = A(\omega)$$

and for any $\alpha \geqq 0$

$$R_k^\alpha(\omega) = \frac{1}{\Gamma(\alpha)}\int_0^\omega (\omega-v)^{\alpha-1}R_k(v)\,dv,$$
$$R_k^0(\omega) = R_k(\omega).$$

If

$$R_k^\alpha(\omega)/\omega^\alpha(\log\omega)^k \to s \text{ as } \omega\to\infty,$$

then we say that $\sum a_n$ be $(\alpha, k)$-summable to the sum $s$ and denote it by

$$\sum a_n = s(\alpha, k).$$

Let $\phi(x)$ be an even periodic function with period $2\pi$, and its Fourier