Journal of the Mathematical Society of Japan Vol. 1, No. 1, Sept., 1948.

On algebraic Lie groups and algebras.

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(Received Oct. 25, 1947.)

Introduction.

Recently C. Chevalley and H. F. Tuan obtained an interesting characterization of the Lie algebras of algebraic Lie groups of matrices[1]. Using the notion of replicas [2] of matrices, they introduced namely the concept of algebraic Lie algebras of matrices ; A Lie algebra $\mathfrak{L}$ of matrices over a field P is called linear algebraic (l-algebraic)[3] if every replica of each $A\in\mathfrak{L}$ belongs also to $\mathfrak{L}$. It was shown by them that, if P is the field of complex numbers, the l-algebraicity is the characteristic property of the Lie algebras of algebraic Lie groups of matrices. The notion has been extended recently by M. Gotô to general, not necessarily matric, Lie algebras[4]. Namely a Lie algcbra is called algebraic if its adjoint representation is l-algebraic. Then he proved that any algebraic Lie algebra over a fiele of characterirtic zero is isomorphic with an l-algebraic Lie algebra of matrices. In this note we shall first prove some results on l-algebraic Lie algebras. Most of these results have been obtained by C. Chevalley and H. F. Tuan, but our methods will be somewhat different from theirs. Then we shall characterize the Lie groups of algebraic Lie algebras over the field of complex numbers. We show that the integrated groups of such Lie algebras are algebraic groups in the sense that the functions which define the multiplication of group elements are algebraic functions of suitably chosen parameters of the group. This result follows also from the above mentioned result of M. Gotô, but our proof is a more direct one. The converse of this proposition has been already proved by L. Maurer[5] and thus we obtain a characterization of the Lie groups of algebraic Lie algebras. The writer is grateful to Mr. M. Gotô for his friendly cooperation.

1. Let P be a field of characteristic zero. For simplicity we call a nilpotent matrix an n-matrix, a matrix with simple elementary divisors an s-matrix, and an s-matrix whose characteristic roots are all rational numbers an r-matrix. Let A be a matrix with coefficients in P and P be algebraically closed. In a previous note[6] we showed that we may represent A in the form