On algebraic Lie algebras.

By Morikuni Goto.

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Introduction.

The classical theory of linear algebraic Lie groups and the associated Lie algebras by L. Maurer\(^1\) has been modernized recently by C. Chevalley and H.-F. Tuan\(^2\) by using the concept of "replica" which was invented by the former of them\(^3\).

Let \(K\) be a field of characteristic 0. A Lie algebra \(\mathfrak{g}\) of matrices over \(K\) is called linear algebraic (l-alg.)\(^4\) if every replica of each matrix \(X\in \mathfrak{g}\) belongs to \(\mathfrak{g}\). If \(K\) is the field of all complex numbers \(C\), any l-alg. Lie algebra is the Lie algebra of a linear algebraic Lie group, and vice versa. This theorem, due to Chevalley and Tuan\(^5\), justifies the definition of l-algebraicity.

The theory of l-alg. Lie algebras has already been established by Chevalley and Tuan. But as their proofs have been reported only in the outline, we shall first give in this note a systematic approach to the theory; our methods will be somewhat different from theirs.

We shall then study the "algebraic closure" (see §4) of any Lie algebra of matrices. Our Theorem 4 is an extension of a theorem of Chevalley and Tuan.

Then we shall give an extension of the theory to, not necessarily matric, algebraic (alg.) Lie algebras. Namely we shall call a Lie algebra alg. if its regular representation is l-alg. Then as we may easily see that any l-alg. Lie algebra is itself alg., our algebraicity is certainly an extension of l-algebraicity. Our fundamental result is given in our Theorem 5, which indicates the complete connection between alg. and l-alg. Lie algebras. And Theorem 6, which is an immediate consequence of our proof of Theorem 5, gives a characterization of an alg. Lie algebra of matrices.

The present study is closely related to recent works of Y. Matsushima on the similar subject and the writer is indebted to him for various discussions\(^6\). In particular Theorem 5 was proved by him independently in the case of Lie algebras over \(C\) by an analytical method using our Lemma 15; the result was used by him to give the characterization of Lie groups