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ON THE LAW OF THE ITERATED LOGARITHM FOR LACUNARY TRIGONOMETRIC SERIES II

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1. Introduction. In this note we set

$$S_N(t) = \sum_1^N a_m \cos 2\pi n_m(t + \alpha_m) \text{ and } A_N = \left(2^{-1} \sum_1^N a_m^2\right)^{1/2},$$

where $a_m \geqq 0$ and $\{n_m\}$ is a sequence of positive integers satisfying the gap condition

(1.1) $\qquad n_{m+1}/n_m \geqq 1 + cm^{-\alpha}$, for some $c > 0$ and $0 \leqq \alpha \leqq 1/2$.

For $\alpha = 0$, M. Weiss [5] proved that if

$$A_N \to +\infty \text{ and } a_N = o(A_N(\log\log A_N)^{-1/2}), \text{ as } N \to +\infty,$$

then for any sequence of $\{\alpha_m\}$

$$\overline{\lim_N}\,(2A_N^2 \log\log A_N)^{-1/2} S_N(t) = 1, \quad \text{a.e.}.$$

For $\alpha > 0$, we proved the following

THEOREM A [4]. *If*

$$A_N \to +\infty \textit{ and } a_N = O(A_N N^{-\alpha}(\log A_N)^{-(1+\varepsilon)/2}), \textit{ as } N \to +\infty,$$

where ε is a positive number, then we have

$$\overline{\lim_N}\,(2A_N^2 \log\log A_N)^{-1/2} S_N(t) \leqq 1, \quad \textit{a.e.}.$$

The purpose of the present note is to prove the

THEOREM B. *Suppose*

(1.2) $\qquad A_N \to +\infty$ *and* $a_N = O(A_N N^{-\alpha}\omega_N^{-1})$, *as* $N \to +\infty$,

where $\omega_N = (\log N)^\beta (\log A_N)^4 + (\log A_N)^8$ and $\beta > 1/2$, then we have

$$\overline{\lim_N}\,(2A_N^2 \log\log A_N)^{-1/2} S_N(t) \geqq 1, \quad \textit{a.e.}.$$

If $\alpha < 1/2$ and $\{a_m\}$ is non-increasing, then by Theorem A and B we obtain