Tôhoku Math. Journ.
27 (1975), 291-301.

ON COMPACT MINIMAL SURFACES WITH NON-NEGATIVE GAUSSIAN CURVATURE IN A SPACE OF CONSTANT CURVATURE: II

KATSUEI KENMOTSU

(Received June 30, 1973)

(Part 1 appeared in the preceding issue of this Journal, Vol. 25)

5. $N_{(b)}$ and $\Omega_{(b)}$ of compact flat surface. As an application of the formulae obtained in the §4, we shall study $N_{(b)}$ and $\Omega_{(b)}$ of a compact flat surface. Let $\bar{M}$ be a space of constant curvature, $c \neq 0$. By the Gauss equation, we have $K_{(2)} = c$ and so $K_{(2)}$ is a positive constant and $c > 0$. Since $f_{(2)}$ is a globally defined non-negative smooth function on M, by $(4.26)_2$, we have $f_{(2)} =$ constant and $A_{(2)} = 0$ on M. By $4N_{(2)} = K_{(2)}^2 - f_{(2)}$, $N_{(2)}$ is also constant on M. By $K_{(2)} > 0$ on M and (3.11), we have $1 \leqq p_1(x) \leqq 2$ at any point of M. Since $N_{(2)}$ is constant, $p_1(x)$ is constant on M. Then the third fundamental forms are defined on a neighborhood of any point of M, i.e., we have $M = \Omega_{(2)}$. If $N_{(2)} = 0$, equivalently, $p_1(x) = 1$ on M, by Lemma 2, there is a 3-dimensional totally geodesic submanifold of $\bar{M}$ such that M is contained in the submanifold as a minimal surface. If $N_{(2)} \neq 0$, then $N_{(2)}$ is a positive constant on M and $p_1(x) = 2$ on M. As $f_{(3)}$ is globally defined on M, by $(4.26)_3$, we have $f_{(3)} =$ constant and $A_{(3)} = 0$. Then we can prove $K_{(3)} =$ constant by virtue of the following Lemma 4 and (4.27).

LEMMA 4. *Let M be a minimal surface in $\bar{M}$. Suppose that*

$$p_a(x) = 2,\ 0 \leqq a \leqq b-2 \text{ and } p_{b-1}(x) = \text{constant on } \Omega_{(b)}\,; \tag{5.1}$$

$$\bar{A}_{(b)} = 0 \text{ on } \Omega_{(b-1)}\,; \tag{5.2}$$

$$K_{(b)} = \text{constant on } \Omega_{(b-1)}\,. \tag{5.3}$$

Then we have

$$N_{(b)} H^{(b)}_{\lambda_{b-1},1} = 0 \text{ on } \Omega_{(b)}\,. \tag{5.4}$$

PROOF. By (5.1), we have $H_\alpha^{(b)} = 0$ for $\alpha \geqq 2b+1$. Then from (4.18) and (5.2), we obtain

$$H^{(b)}_{(2b-1)} H^{(b)}_{(2b-1),1} + H^{(b)}_{(2b)} H^{(b)}_{(2b),1} = 0\,. \tag{5.5}$$

Since $K_{(b)} =$ constant and (4.24), we get