SOME REMARKS CONCERNING PRINCIPAL IDEAL THEOREM*)

By

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1.

In this paper I present a detailed account of the results previously announced in the Proceedings of the Academy of Tokyo, and which constitute the arithmetic complement to Terada's paper in this same volume. Concerning the historical note we refer to the above previous note and the preface in Terada's paper.

In the paragraph 6 of this paper I give a proof of original principal ideal theorem, which is in substance that of Iyanaga's paper [1], but which does not depend on the concept of "order ideal". § 7 contains further remark in this direction.

2.

Mr. Terada has proved recently the following generalization of Furtwängler's principal ideal theorem:

THEOREM 1. *Let K be the absolute class field over k, and Ω a cyclic intermediate field of K/k, then all the ambigous ideal classes of Ω will become principal in K.*

This Theorem was suggested by the special case $K=\Omega$, where no essential difficulty occurs. We can prove that special case for instance by the principal genus theorem, which asserts that $N_{K/k}\mathfrak{a}\sim 1$ implies $\mathfrak{a}\sim\mathfrak{b}^{1-s}$ for some ideal $\mathfrak{b}$, s being a generator of the Galois group $G(K/k)$. We see namely that in our case the correspondence $\mathfrak{a}\rightarrow\mathfrak{a}^{1-s}$ leads to an isomorphism of the ideal classes, and the required proof is at hand.

I will show in the following line in what manner Iyanaga's principal ideal theorem [2] for "ray class fields" (Strahlklassenkörper) can be generalized, and that this amounts to

THEOREM 2. *Let K be the ray class field mod $\mathfrak{f}(K/k)$ over k, and Ω a cyclic intermediate field of K/k. Let also $\mathfrak{m}$ denote the ideal Max $\{\mathfrak{f}(K/\Omega), \mathfrak{F}(\Omega/k)\}$ in Ω. If $\mathfrak{a}$ is an ideal in ambigous class modulo $\mathfrak{m}$, then $\mathfrak{a}$ lies in the ray modulo $\mathfrak{F}(K/k)$, when considered as an ideal in K. Thereby*

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