NOTES ON BANACH SPACE (XII): A REMARK ON A THEOREM OF GELFAND AND NEUMARK*)

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Following C.E.Rickart [7], a Banach algebra R over complex numbers having a principal unit 1 is called a *star-algebra* if it has an operation x^* satisfying

$$1^\circ \quad (\lambda x + \mu y)^* = \lambda^* x^* + \mu^* y^*,$$
$$2^\circ \quad (xy)^* = y^* x^*,$$
$$3^\circ \quad x^{**} = x,$$
$$4^\circ \quad |xx^*| = |x|^2.$$

It is reported by the American literatures, although the original paper of I. Gelfand and M. Neumark [3] is not yet available in this country, that they have proved in 1943 that a commutative star-algebra is isometrically isomorphic with the algebra of all complex-valued functions on a compact Hausdorff space. Recently, R. Arens [1] simplified and clarified the proof of this theorem. In the general case, it is also reported, they have proved that the algebra is isometrically ismorphic with a ring of operators on a certain Hilbert space under an additional assumption, which states that $1 + xx^*$ has an inverse in R for any x.

The purpose of the present note is to show, that the later theorem is also t ue when the above condition is replaced by another one, and that the theorem is proved in a similar manner as in that of I. E. Sagal [8]. In the below, to save the spice, it is assumed that the readers are familiar with the stir-algebras, and so we will only describe the outline of the proofs when they are already known.

1. In this section, we may prove some geneneral properties of star algebras. The essential materials are taken from I. E. Segal [8].

Definition 1. A linear functional f on R is said to be *positive*, symbolically $f \geqq 0$, if $f(xx^*) \geqq 0$ for any x of R. A positive linear functional is

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