

ON THE SPACES WITH NORMAL CONFORMAL CONNEXIONS AND SOME IMBEDDING PROBLEM OF RIEMANNIAN SPACES, II*)

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In the previous paper¹⁾ we have studied the spaces with normal conformal connexions whose groups of holonomy fix a point or a hypersphere. The main results that we have obtained are as follows:

(1) If the group of holonomy of an $(n+1)$ -dimensional space C_{n+1} with a normal conformal connexion fixes a hypersphere \mathfrak{S}_n , the C_{n+1} is a space with a normal conformal connexion corresponding to the class of Riemannian spaces conformal to each other including an Einstein space with a negative, vanishing or positive scalar curvature according as the \mathfrak{S}_n is real, point or imaginary. The converse is also true.

(2) For $n = 2m + 1$ ($m \geq 1$) and 2 any Riemannian space V_n , and for $n = 2m$ ($m \geq 2$) any Riemannian space V_n satisfying the condition $L_\lambda^\nu = 0$ can be imbedded in a Riemannian space V_{n+1} conformal with some Einstein space as a hypersurface which is the image of a hypersphere \mathfrak{S}_n invariant under the group of holonomy of the space C_{n+1} with the normal conformal connexion associated with this V_{n+1} .

But the meaning of the immersion of a given Riemannian space V_n in a V_{n+1} as a hypersurface of it as stated above is that at each point P of V_{n+1} , the invariant hypersphere \mathfrak{S}_n in the tangent Möbius space $M_{n+1}(P)$ at P under the group of holonomy of C_{n+1} *contain the point at infinity in $M_{n+1}(P)$* (with respect to the natural frame of C_{n+1}), and the image of \mathfrak{S}_n in V_{n+1} is the set of points P such that P as a point in $M_{n+1}(P)$ is contained in \mathfrak{S}_n .

In the present paper, we shall investigate the same problem to imbed a given Riemannian space V_n in an V_{n+1} as stated above *without the restriction such that \mathfrak{S}_n contains the point at infinity in the tangent Möbius space $M_{n+1}(P)$ at each point P of V_{n+1}* , in other words, without any restriction with respect to the scalar y^0 (in the previous paper, no. 1, 2).

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1) Tominosuke Ôtsuki, On the spaces with normal conformal connexions and some imbedding problem of Riemannian spaces, I, Tohoku Math. Jour., 2nd. S., Vol. 1, No. 2, 1950, pp. 194-224. We shall refer this paper by [I] in the present paper.