A CLASS OF SINGULAR INTEGRAL EQUATIONS

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Some years ago, Professor K. Kondô proposed to solve the following integral equation

$$(1) \qquad f(y) = \frac{1}{\pi}(P)\int_{-1}^{1}\frac{u(x)}{x-y}\,dx - \frac{1}{\pi}\int_{-1}^{1}k(y,x)u(x)dx, \quad (-1<y<1)$$

where (P) indicates Cauchy's principal value. This equation is related to a problem of aerodynamics. The case $k(x,y)\equiv 0$ was first solved by Fuchs-Hopf-Seewald (cf. G. Hamel [1] p. 145) employing Fourier series expansions. Subsequently K. Schröder [3] gave a beautiful solution with the aid of conjugate functions. The author [4] have ever given a solution of (1) following to Schlöder's method. Recently we have been able to see some periodicals published during the war, and learned that Professor E. Reissner [2] has already solved an integral equation of the type (1). But his method seems to be analogous to Fuchs-Hopf-Seewald, and the solution is given by its Fourier series. In this paper we give a method to solve the equation (1), which is different from that of Reissner and gives an easier solution in some cases.

We shall begin with a reciprocal formula of conjugate functions. Let $H(\theta)$ be even and $G(\theta)$ be odd, then we have

$$(2) \qquad G(\theta) = -\frac{1}{\pi}(P)\int_{0}^{\pi}H(\varphi)\frac{\sin\theta}{\cos\varphi-\cos\theta}\,d\varphi,$$

and

$$(3) \qquad H(\theta) = -\frac{1}{\pi}(P)\int_{0}^{\pi}G(\varphi)\frac{\sin\varphi}{\cos\theta-\cos\varphi}\,d\varphi + c,$$

where

$$c = \frac{1}{\pi}\int_{0}^{\pi}H(\varphi)d\varphi$$

(see Schröder [3]). Especially, if $H(\varphi)$ and $G(\varphi)$ belong to $L^2(0,\pi)$, then reciprocal formula (2) and (3) is valid almost everywhere.

If we put, in (1),

$$(4) \qquad y=\cos\theta\,(0\leqq\theta\leqq\pi) \text{ and } x=\cos\varphi\,(0\leqq\varphi\leqq\pi),$$

then we have

$$f(\cos\theta) = -\frac{1}{\pi}(P)\int_{\pi}^{0}\frac{u(\cos\varphi)\sin\varphi}{\cos\varphi-\cos\theta}\,d\varphi - \frac{1}{\pi}\int_{\pi}^{0}u(\cos\varphi)\,k(\cos\theta,\ \cos\varphi)\sin\varphi\,d\varphi,$$