AN EXTENSION OF BLOCH'S THEOREM AND ITS APPLICATIONS TO NORMAL FAMILY

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In the former paper[1], I have proved the following extension of Bloch's theorem :

THEOREM 1. *Let $w = f(z)$ be meromorphic in $|z| < 1$ and*

$$\frac{|f'(0)|}{1 + |f(0)|^2} = 1.$$

Then the Riemann surface F generated by $w = f(z)$ on the w-sphere contains a schlicht spherical disc, whose radius is $\geqq \rho_0 > 0$, where ρ_0 is a constant independent of $f(z)$.

In this paper, I shall apply this theorem to normal family.

THEOREM 2.[2] *Let $D_1, \cdots, D_q$ $(q \geqq 3)$ be q disjoint simply connected domains on the w-sphere and $1 \leqq m_i \leqq \infty$ be positive integers or ∞, such that*

$$\sum_{i=1}^{q}(1 - 1/m_i) > 2.$$

Let $w = f(z)$ be mermorphic in $|z| < R$ and F be the Riemann surface generated by $w = f(z)$ on the w-sphere. If every simply connected island of F, which lies above D_i is of multiplicity $\geqq m_i$[3]*, then*

$$R \leqq \kappa \frac{1 + |f(0)|^2}{|f'(0)|}, \qquad \frac{|f(0)|}{1 + |f(0)|^2} \leqq \frac{\kappa}{R},$$

where κ is a constant, which depends on $D_1, \cdots, D_q$ only.

PROOF. It can be proved easily that if $\sum_{i=1}^{q}(1 - 1/m_i) > 2$, then

$$\sum_{i=1}^{q}(1 - 1/m_i) - 2 \geqq 1/42, \tag{1}$$

where the minimum value $1/42$ is attained, when $m_1 = 2$, $m_2 = 3$, $m_3 = 7$, $m_4 = \cdots = m_q = 1$.

First suppose that

$$\frac{|f'(0)|}{1 + |f(0)|^2} = 1. \tag{2}$$

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1) M. TSUJI, On an extension of Bloch's theorem. Proc. Imp. Acad., 18(1942).

2) J. DUFRESNOY, Sur les domaines couverts par les valeurs d'une fonction méromorphe ou algébroïde. (Thèse, 1935). Z. YÛJÔBÔ, An application of Ahlfors' theory of covering surfaces. Jour. Math. Soc. Japan., 4(1952).

3) $m_i = \infty$ means that there is no island of F above D_i.