ON ABSOLUTE LOGARITHMIC SUMMABILITY OF A SEQUENCE RELATED TO A FOURIER SERIES

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(Received January, 10, 1954)

1. DEFINITION A. Let $\lambda(\omega)$ be continuous, differentiable and monotone increasing in (A, ∞), where A is some positive number, and let $\lambda(\omega) \to \infty$ as $\omega \to \infty$. Suppose Σu_n is a given infinite series and let

$$c(\omega) = \sum_{n \leqq \omega} \{\lambda(\omega) - \lambda(n)\} u_n.$$

The series Σu_n is said to be summable $|R, \lambda(n), 1|$ if

$$\int_A^\infty \left| d\left[\frac{c(\omega)}{\lambda(\omega)}\right]\right| < \infty,$$

i.e. if

$$\int_A^\infty \frac{\lambda'(\omega)}{\{\lambda(\omega)\}^2} \left| \sum_{n \leqq \omega} \lambda(n) u_n \right| d\omega < \infty.$$

DEFINITION B. Suppose $\{t_n\}$ is a given sequence and let $\tau_n = \left(t_1 + \frac{1}{2}t_2 + \dots + \frac{1}{n}t_n\right)/\log n$. If $\tau_n \to t$ as $n \to \infty$, then the sequence $\{t_n\}$ is said to be summable $(R, \log n, 1)$ to t. If the sequence $\{\tau_n\}$ is of bounded variation, i.e., if $\sum^\infty |\tau_n - \tau_{n+1}| < \infty$, the sequence is said to be summable $|R, \log n, 1|$.

2. Let $\varphi(t)$ be an even function integrable in the sense of Lebesgue in $(0, \pi)$ and defined outside $(-\pi, \pi)$ by periodicity. We assume that the constant term in the Fourier series of $\varphi(t)$ is zero and that the special point to be considered is the origin. In these circumstances

$$\varphi(t) \sim \sum_1^\infty a_n \cos nt, \tag{2.1}$$

where

$$a_n = \frac{2}{\pi} \int_0^\pi \varphi(t) \cos nt \, dt, \tag{2.2}$$

and we are to consider the series $\sum_1^\infty a_n$. It is well-known that these formal simplifications do not impair the generality of the problem. We write s_n for $\sum_1^n a_k$ and use the following notations.