ON FUNCTIONS REGULAR IN A HALF-PLANE

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1. Let $\varphi(z)$ be an analytic function, regular for $y > 0$, and let

$$\int_{-\infty}^{\infty} |\varphi(x+iy)|^p \, dx \leqq K^{*)} \qquad (p > 0)$$

for all value $y > 0$. Then we say that $\varphi(z)$ belongs to the class $\mathfrak{H}_p$ (= The Hille-Tamarkin Class). E. Hille and J. D. Tamarkin [2], for $p \geqq 1$ and T. Kawata [3], for $1 > p > 0$, proved the following theorems.

THEOREM A. (1) *A function $\varphi(z) \in \mathfrak{H}_p$ tends to a limit function $\varphi(x)$ in the mean of order p, and*

$$\int_{-\infty}^{\infty} |\varphi(x+iy)|^p \, dx \uparrow \int_{-\infty}^{\infty} |\varphi(x)|^p \, dx \qquad \text{as } y \downarrow 0.$$

(2) *Any $\varphi(z) \in \mathfrak{H}_p$ for almost all x tends to its limit function $\varphi(x)$ along any non-tangential path.*

THEOREM B. *A function $\varphi(z) \in \mathfrak{H}_p$ can be represented as a product $\varphi(z) = B(z)\psi(z)$ where $B(z)$ is the Blaschke product and $\psi(z) \in \mathfrak{H}_p$ which does not vanish in $y > 0$.*

THEOREM C. *If the limit function $\varphi(x) \in L_p$, $1 \leqq p \leqq \infty$ has a Fourier transform $\Phi(x)$ in L_q $(1 \leqq q \leqq \infty)$, then the Poisson integral associated with $\varphi(x)$ can be written in the form*

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \varphi(t) \frac{y\,dt}{(t-x)^2+y^2} = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{ixt} e^{-yt} \Phi(t)\,dt.$$

These theorems are counterparts of theorems on functions belonging to class H_p $(p > 0)$ in a unit circle. Recently D. Waterman [6] proved $\mathfrak{H}_p$ $(p > 1)$ analogue of the Littlewood-Paley and Zygmund theorems. In the present note, the author shows some generalized theorems following on his former paper [5].

We put by the definition

$$g_\alpha^*(x) \equiv g_\alpha^*(x;\varphi) = \left\{ \frac{1}{\pi} \int_0^{\infty} y^{2\alpha}\, dy \int_{-\infty}^{\infty} \frac{|\varphi'(t+iy)|^2}{|t-z|^{2\alpha}}\, dt \right\}^{1/2}$$

*) Throughout this paper, $A, B \ldots\ldots$ are constants and may be different from one occurence to another.