ON THE CESÀRO SUMMABILITY OF FOURIER SERIES (III)

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1. Let $\varphi(t)$ be an even integrable function with period 2π and let

$$\varphi(t) \sim \sum_{n=1}^{\infty} a_n \cos nt, \qquad a_0 = 0, \tag{1.1}$$

$$\varphi_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \varphi(u)(u-t)^{\alpha-1}\, du \quad (\alpha > 0), \tag{1.2}$$

and S_n^β be the β-th Cesàro sum of the Fourier series of $\varphi(t)$ at $t = 0$, that is,

$$S_n^\beta = \sum_{\nu=0}^{n} A_{n-\nu}^\beta a_\nu \qquad (\beta > -1). \tag{1.3}$$

C. T. Loo [7] proved the following theorem.

THEOREM A. *If* $\alpha > 0$ *and*

$$S_n^\alpha = o(n^\alpha / \log n) \qquad \textit{as } n \to \infty, \tag{1.4}$$

then

$$\varphi_{1+\alpha}(t) = o(t^{1+\alpha}).$$

This theorem is the converse type of Izumi-Sunouchi's theorem [5]. Recently, we proved the following theorem [6]:

THEOREM B. *If*

$$\varphi_\beta(t) = o\left\{ t^\beta \Big/ \left(\log \frac{1}{t}\right)^{\frac{1}{\gamma}} \right\} \qquad (\beta, \gamma > 0) \qquad \textit{as } t \to 0,$$

and

$$\int_0^t \left| d\left\{ \frac{t\varphi(t)}{\left(\log \frac{1}{t}\right)^\Delta} \right\} \right| = O(t) \qquad (\Delta > 0,\ 0 < t \leqq \eta),$$

then

$$S_n^\alpha = o(n^\alpha),$$

where $\quad \alpha = (\Delta\gamma\beta - 1)/(1 + \Delta\gamma).$

In the present note we prove a theorem which is the converse type of theorem B.

THEOREM. *If*

$$a_n > -K(\log n)^\alpha / n \qquad (\alpha > 0) \qquad \textit{as } n \to \infty \tag{1.5}$$