ON THE INTERPOLATION OF ANALYTIC FAMILIES OF OPERATORS ACTING ON H^p-SPACES

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1. Introduction. Let $\mathfrak{P}$ be the class of all polynomials $P(w) = a_0 + a_1 w + \ldots + a_k w^k$, where the a_j's $(j = 1, 2, \ldots, k)$ are complex numbers and w is a complex variable. If $p > 0$ we can form the space H^p, [11], of all functions $F(w)$, analytic in the interior of the unit circle, satisfying

$$\mu_p(r\,;F) = \frac{1}{2\pi}\int_{-\pi}^{\pi} |F(re^{i\theta})|^p d\theta \leqq M < \infty, \tag{1.1}$$

where M is independent of r, $0 \leqq r < 1$. It is well known that $\mu_p(r\,;F)$ is a non-decreasing function of r, and, if $p \geqq 1$,

$$\|F\|_p = \{\lim_{r\to 1} \mu_p(r\,;F)\}^{1/p} \tag{1.2}$$

is a norm. In fact, with this norm, H^p is complete; i. e. it is a Banach space. In case $p < 1$, however, $\|F\|_p$ is not a norm since the triangle inequality is no longer satisfied. [1] H^p, nevertheless, can be made into a complete topological vector space by introducing the metric $d_p(F, G) = \|F - G\|_p^p$. In either of these cases the class $\mathfrak{P}$ is dense in $H^p, p > 0$. We will make repeated use of the fact that, if $0 < p_1 \leqq p_2$, then $\|F\|_{p_1} \leqq \|F\|_{p_2}$.

Let (M, μ) be a measure space, where M is the point set and μ the measure. If $q > 0$, $L^q(M, \mu) = L_q$ will denote the space of all complex-valued measurable functions, f, defined on M such that

$$\|f\|_q = \left\{\int_M |f|^q d\mu\right\}^{1/q} < \infty. \tag{1.3}$$

We will refer to $\|f\|_q$ as the norm of f. Remarks analogous to the ones made about $\|F\|_p$ apply here: if $q \geqq 1$, L^q becomes a Banach space; while, if $q < 1$, $d_q(f, g) = \|f - g\|_q^q$ is a metric. [2]

We say that a linear transformation, T, mapping $\mathfrak{P}$ into a class of measurable functions defined on M is of type (p, q) in case there exists a constant $A > 0$ such that

$$\|TP\|_q \leqq A\|P\|_p, \tag{1.4}$$

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1) In order to avoid introducing unnecessary terminology, we shall still refer to $\|F\|_p$ as "the norm of F" when $p<1$.

2) No confusion should arise from the fact that we use the same [notation for the H^p-norm and the L^q-norm.