A SUFFICIENT CONDITION FOR THE ABSOLUTE RIESZ SUMMABILITY OF A FOURIER SERIES

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1. We suppose that $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$, and we write

$$f(t) \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=0}^{\infty} A_n(t),$$

$$\phi(t) = \frac{1}{2} \{f(t+x) + f(x-t)\},$$

$$\Phi_\alpha(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-u)^{\alpha-1} \phi(u)\, du \ (\alpha > 0), \quad \phi_\alpha(t) = \Gamma(\alpha+1) t^{-\alpha} \Phi_\alpha(t) \ (\alpha > 0),$$

Concerning the absolute Riesz summability $|R, \lambda(w), k|$, where the type $\lambda(w)$ is equal to $\exp\{(\log w)^\Delta\}$, $(\Delta > 1)$, the following theorems are known.

THEOREM. Mohanty and Misra [1], Kinukawa [2]. *If* $\phi_\alpha(t) \log(k/t)$, *where* $k > \pi e^2$, *is of bounded variation in* $(0, \pi)$, *then the series* $\sum_2^\infty A_n(x)$ *is summable* $|R, \exp\{(\log w)^\Delta\}, 1|$, *where* $0 < \alpha < 1$ *and* $\Delta = 1 + 1/\alpha$.

THEOREM. Pati [3]. *If* α *is an integer* $\geqq 1$, *and* $\phi_\alpha(t) \log(k/t)$, $(k > \pi e^{\alpha+2})$ *is of bounded variation in* $(0, \pi)$, *then the Fouries series of* $f(t)$, *at* $t = x$, *is summable* $|R, \exp\{(\log w)^{1+1/\alpha}\}, 1 + \alpha|$.

We shall prove here the following

THEOREM[1]. *If* $\phi_\alpha(t)(\log k/t)^{\alpha(\Delta-1)}$, $(k > \pi e^{\alpha(\Delta-1)+1})$, *is of bounded variation in* $(0, \pi)$, *then* $\sum_{n=0}^{\infty} A_n(x)$ *is summable* $|R, \exp\{(\log w)^\Delta\}, \beta|$, *where* $\beta > \alpha > 0$ *and* $\Delta \geqq 1$.

This theorem is an improvement of the above two theorems, and when $\Delta = 1$ this theorem reduces to a theorem on $|C, k|$ summability proved by L. S. Bosanquet [4], further this theorem shows that the summability $|R, \exp\{(\log w)^\Delta\}, \beta|$, $\beta > 1$, of a Fourier series is a local property of the generating function.

For the proof of the theorem, it suffices to show that, when $\Delta > 1$,

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1). The $(R, \exp\{(\log w)^\Delta\}, \beta)$-analogue has already been proved by K. Kanno On the Riesz summability of Fourier series, Tôhoku Math. Journ., 8(1956), in somewhat weak form. But we can complete the Kanno theorem, applying the method of this paper.