COHOMOLOGY THEORY AND DIFFERENT

HIDEO KUNIYOSHI

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The relations between cohomology groups and different in the number theory were already treated by A. Weil [11], Y. Kawada [6], A. Kinohara [7] and M. Moriya [9] in cases of dimension 1 and 2. In the present paper we shall treat the same subjects for general dimensions under a slight modification.

In § 1 we shall explain the definitions and main results of this note. In § 2 we shall prove the equalities of the right-, left- and two sided homological differents. § 3 and § 4 are preliminaries for the following sections. In § 5 we shall prove, essentially, that the homological different is not zero, and in § 6 we shall treat the reduction to the local homological different. In § 7 we shall consider the local homological different and prove the different theorem, and in § 8 we shall show the equality between homological differents and the usual different.

1. Definitions and results. Let R be a Dedekind ring, K its quotient field, L a finite separable extension field over K and Λ the principal order (the unique maximal order) of L over R. We regard Λ as an algebra over R. [1] For any two sided Λ-module A, the homology groups $H_n(\Lambda, A)$ and the cohomology groups $H^n(\Lambda, A)$ are defined as usual [1] i.e.

$$H_n(\Lambda, A) = \mathrm{Tor}_n^{\Lambda^e}(A, \Lambda), \qquad \text{(1.1)}$$
$$H^n(\Lambda, A) = \mathrm{Ext}_{\Lambda^e}^n(\Lambda, A).$$

An element $\lambda^e = \Sigma \lambda \otimes \mu$ of Λ^e induces a Λ^e-endomorphism $\overline{\lambda^e}$ of A

$$\text{(1.2)} \qquad \overline{\lambda^e} : A \to A, \quad \overline{\lambda^e}(a) = \lambda^e a\,;$$

$\overline{\lambda^e}$ induces an endomorphism $\widetilde{\lambda^e}$ of $H(\Lambda, A)$

$$\widetilde{\lambda^e} : H_n(\Lambda, A) \to H_n(\Lambda, A), \qquad \text{(1.3)}$$
$$H_n(\Lambda, A) \to H_n(\Lambda, A).$$

Therefore $H(\Lambda, A)$ may be considered as a Λ^e-module. Using these endomorphisms $\widetilde{\lambda^e}$, we define the n-homological (cohomological) different of Λ/R.

DEFINITION 1. Left n-homological and cohomological differents $D_n^l(\Lambda/R)$ and $D_l^n(\Lambda/R)$:

$$D_n^l(\Lambda/R) = \{\lambda \in \Lambda \mid \widetilde{\lambda \otimes 1}\, H_n(\Lambda, A) = 0 \quad \text{for all } A\},$$
$$D_l^n(\Lambda/R) = \{\lambda \in \Lambda \mid \widetilde{\lambda \otimes 1}\, H^n(\Lambda, A) = 0 \quad \text{for all } A\}.$$

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1) In the following our main objects are these algebras, which we shall quote as "the number theoretical algebras" or "the number theoretical cases".