ON DETERMINATION OF THE CLASS OF SATURATION IN THE THEORY OF APPROXIMATION OF FUNCTIONS II

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1. Introduction. Let $f(x)$ be an integrable function, with period 2π and let its Fourier series be

$$\mathfrak{S}[f] \equiv \sum_{k=0}^{\infty} A_k(x) \equiv \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) \tag{1}$$

Let $g_k(n)$ $(k = 0, 1, 2, \ldots)$, $g_0(n) = 1$ be the "summating" function and consider a family of transforms of (1) by a method of summation G

$$P_n(x) = \sum_{k=0}^{\infty} g_k(n) A_k(x) \tag{2}$$

where the parameter n need not be discrete.

If there are a positive non-increasing function $\varphi(n)$ and a class K of functions in such a way that

(I) $\quad \|f(x) - P_n(x)\| = o(\varphi(n))$ implies $f(x) =$ constant;

(II) $\quad \|f(x) - P_n(x)\| = O(\varphi(n))$ implies $f(x) \in K$

(III) $\quad f(x) \in K$ implies $\|f(x) - P_n(x)\| = O(\varphi(n))$

then it is said that the method of summation G is saturated with the order $\varphi(n)$ and with the class of saturation K.

Since the above definition was given by J. Favard [3], a number of authors have published their result: G. Alexits [0], P. L. Butzer, [2], J. Favard himself [4], M. Zamansky [9] and others.

The purpose of the present paper lies in giving proofs to the theorems stated in our previous paper [8].

Throughout the paper the norms should be taken *with respect to the variable* x, and the subscript p to L^p-norms will generally be omitted. Another convention is that the space (C) is meant the notation L^∞. (or, the case $p = \infty$ of L^p). Thus the generalized Minkowski inequality reads

$$\left\| \int f(x,t)dt \right\| \leqq \int \|f(x,t)\|\, dt, \qquad p \geqq 1$$

and the class $\mathrm{Lip}(\alpha, p)$ with $p = \infty$ reduces to Lip α.

2. The inverse problem. Let us write $\Delta_n(x) = f(x) - P_n(x)$ and suppose that for some positive function $\psi(k)$ and a positive constant c, we have