THE STRUCTURE OF A RIEMANNIAN MANIFOLD ADMITTING A PARALLEL FIELD OF TANGENT VECTOR SUBSPACES

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1. Introduction. Let M be an n-dimensional connected complete Riemannian manifold of class C^2 admitting a parallel field of r-dimensional tangent vector subspaces. Then, M admits the parallel field of s-dimensional tangent vector subspaces, where $s = n - r$, orthogonal to the given field. M is also regarded as a Riemannian manifold whose homogeneous holonomy group fixes an r- (or s-) dimensional tangent vector subspace. The purpose of this note is to treat of the global structure of M. In a case where $r = n - 1$, i.e. $s = 1$, the author [3] already attempted to clarify geometrically the global structure. Here let us discuss the structure in the case where $1 \leqq r$, $s \leqq n - 1$, from the view-point of fibre bundle. For the main results, see Theorems 1—7. Especially Theorem 3 shows a general structure of M and from the other theorems we may know structures in respective cases. Notice that these theorems all hold good even if R and S in these theorems are exchanged for each other (see Remark 1).

From now on the word "k-dimensional" is abbreviated as "k-", say like k-space (but, such a prefix does not necessarily mean dimension). Let us suppose that indices run as follows: $a, b = 1, 2, \ldots, r$; $i, j = r + 1, r + 2, \ldots, n$; $\alpha = 1, 2, \ldots, n$. The following conventions in a Riemannian manifold X are also applied to all of Riemannian manifolds: The *parallelism* in X means the one of Levi-Civita. A *neighborhood* in X is always an open set homeomorphic to Euclidean space. Take any $x, y \in X$. Let $[x, y]$ denote a geodesic arc joining x to y. And further, take a unit tangent vector v at x. Given a real number c, $g(x, v, c)$ is defined to be the geodesic arc issuing from x, whose length is $|c|$ and whose initial vector is v or $-v$ according as $c > 0$ or < 0. Let (x, v, c) denote its terminal point. Note that a geodesic arc is not necessarily simple and sometimes may be closed. Let a curve $\alpha : x(t)$ (say, $0 \leqq t \leqq 1$) be given in X. At $x_0 = x(0)$ we take a unit vector v_0 tangent to X. Corresponding to each t, let $v(t)$ denote the unit vector at $x(t)$ parallel to v_0 along α. Moreover, if a geodesic arc $g(x_0, v_0, c)$ is given, each geodesic arc $g(x(t), v(t), c)$ is said to be *parallel* to $g(x_0, v_0, c)$ along α. And to displace the latter arc parallelly along α is to obtain the former arcs. A *covering manifold* $C(X)$ of X is defined to be a connected covering manifold of X