ON THE ABSOLUTE SUMMABILITY FACTORS OF INFINITE SERIES I

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1. 1. Definitions. Let $\sum a_n$ be a given infinite series, and let s_n^α and t_n^α denote the n-th Cesàro means of order α $(\alpha > -1)$ of the sequences $\{s_n\}$ and $\{na_n\}$ respectively, where s_n is the n-th partial sum. The series $\sum a_n$ is said to be absolutely summable (C, α), or summable $|C, \alpha|$, if the sequence $\{s_n^\alpha\}$ is of bounded variation, that is, if the infinite series $\sum |s_n^\alpha - s_{n-1}^\alpha|$ is convergent ([4], [6]).

1. 2. In what follows we shall require the following identities.

(1. 2. 1) $$t_n^\alpha = n(s_n^\alpha - s_{n-1}^\alpha) \qquad ([6], [7]);$$

(1. 2. 2) $$t_n^\alpha = \frac{1}{A_n^\alpha} \sum_{\nu=1}^{n} A_{n-\nu}^{\alpha-1} \nu a_\nu,$$

where

(1. 2. 3) $$\sum_{n=0}^{\infty} A_n^\alpha x^n = (1 - x)^{-\alpha-1} \qquad (|x| < 1);$$

and, by definition ([5]),

(1. 2. 4) $$A_{-1}^\alpha = 0,\ A_0^{-1} = 1,\ A_n^{-1} = 0 \quad (n \geqq 1);\ A_n^{-2} = 0 \quad (n \geqq 2);$$

(1. 2. 5) $$A_n^\alpha = \begin{cases} \binom{n+\alpha}{n} & (\alpha > -1), \\ (-1)^n \binom{-\alpha-1}{n} & (\alpha \leqq -1); \end{cases}$$

(1. 2. 6) $$A_n^\alpha = \Gamma(n + \alpha + 1)/\{\Gamma(n + 1)\Gamma(\alpha + 1)\}$$
$$\sim n^\alpha/\Gamma(\alpha + 1) \ (\alpha \neq -1, -2, \ldots\ldots\ldots).$$

For any sequence $\{\varepsilon_n\}$, we write

(1. 2. 7) $$\Delta^0\varepsilon_n = \varepsilon_n,\ \Delta\varepsilon_n = \Delta^1\varepsilon_n = \varepsilon_n - \varepsilon_{n+1},$$

and