TRANSFORMATIONS OF CONJUGATE FUNCTIONS

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1. Throughout this paper we suppose that each function is defined in $(-\pi,\pi)$, integrable and is periodic with period 2π. If a function $W(x)$, which is defined in $(0,\pi)$, is extended to $(-\pi,\pi)$ as an even function, we denote it by $W_c(x)$, and if extended as an odd function, we denote it by $W_s(x)$. For a given function $f(x)$, we shall denote its conjugate function by $\overline{f}(x)$.

We shall be concerned with the following types of transformations T_H and T_H^* of a function $f(x)$;

$$T_H f(x) = \int_x^\pi \frac{f(t)}{2\tan t/2}\,dt \equiv F(x)$$

and

$$T_H^* f(x) = \frac{1}{2\tan x/2}\int_0^x f(t)dt \equiv F^*(x).$$

These transformations were discussed explicitly or implicitly by Bellman [1], Boas and Izumi [2], Hardy [4], Kawata [6], Loo [7] and Sunouchi [8]. Here we have the following results:

(1) *If $g(x)$ is odd and integrable, then*

$$\overline{G_s}(x) = \overline{(T_H g)_s}(x)$$

is also integrable in $(-\pi,\pi)$;

(ii) *If* $\int_0^\pi |g(x)|(\log^+ 1/x)dx < \infty$, *then*

$$\overline{G_s^*}(x) = \overline{(T_H^* g)_s}(x)$$

is integrable.

These results, which are somewhat better than Loo's theorems ([7], Theorems 4 and 7), can be proved easily by using the following lemma due to

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