AN APPLICATION OF A METHOD OF MARCINKIEWICZ TO THE ABSOLUTE SUMMABILITY OF FOURIER SERIES

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1. Introduction. Various sufficient conditions are known for the convergence of Fourier series at an assigned point or almost everywhere. For example, the Dini, Lebesgue and other tests for the former case and the Kolmogoroff-Seliverstoff and the Marcinkiewicz theorems for the "almost everywhere" case. On the other hand Marcinkiewicz obtained a theorem of another type, that is, for an integrable function $f(x)$ of period 2π, if

$$\int_0^h |f(x+t)-f(x)|\,dt = O(|h|/\log|1/h|)$$

as $h \to 0$ for every x in a set E, then its Fourier series converges almost everywhere in E.

A proof of this theorem is based on a decomposition of the function $f(x)$ into the sum of two functions $\psi(x)$ and $\phi(x)$ which satisfy the Dini and the Kolmogoroff-Seliverstoff conditions for some values of x respectively, and on the localization property of convergence of Fourier series.

In this note we are interested in an application of his method to the absolute summability of Fourier series which has the localization property for suitable summability indices.

2. For the absolute summability the following theorems are known.

THEOERM A ([1]). *Let $1 \leqq p \leqq 2$ and let $f(x)$ be of period 2π and integrable $L^p(0, 2\pi)$. If the series*

$$\sum_{j=0}^{\infty}\left(\int_{\pi/2^{j+1}}^{\pi/2^j} \frac{|\varphi_x(t)|^p}{t}\,dt\right)^{1/p} \tag{2.1}$$

where $2\varphi_x(t) = f(x+t)+f(x-t)-2f(x)$, is convergent, then the Fourier series of f is summable $|C, 1/p+\varepsilon|$ at the point x for any $\varepsilon > 0$.

This theorem shows that the summability $|C, 1/p+\varepsilon|$ is of local property for $1 \leqq p \leqq 2$. We note that the convergence of the series (2.1) is easily obtained from each of the next two conditions: