EXTENSIONS OF RINGS OF OPERATORS ON HILBERT SPACES

NOBORU SUZUKI

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Introduction. In the development of the extension theory of rings of operators on a Hilbert space, the study of the direct product of factors[1] initiated by Murray and von Neumann [6], [7] has been developed successively for general rings of operators by Dixmier [1] and Misonou [5]. In recent years, we have brought the notion of the crossed product of division algebras or simple algebras into rings of operators, more particularly, finite factors and established the foundation of the theory of the crossed product. Subsequently it has been examined by Saitô [10], [12] and the other authors. The initial impetus which led us to these investigation was provided by the so-called factor construction created by Murray and von Neuman [6], [8]. Making use of the crossed product, we have seen that the factors of different algebraical types from the original one are constructed by varying the groups of automorphisms[2] [11], [14].

The purpose of this paper is to present a unified account of these development and to study the more general types of extensions of finite factors. Let **A** be a finite factor on a Hilbert space, then a factor **M** containing **A** as a subfactor is called an extension of **A**. The first class of the extension **M** of **A** which is called the discrete extension was indicated by the classification of the dimension type for rings of operators, in particular, the class of type I. In the first step it will be shown that the discrete extension involves not only the $n \times n$ matrix algebra over **A**, but also the crossed product of **A** by a certain group of automorphisms.

The second class of the extension **M** which is called the splitting extension of **A** by a group G is, roughly speaking, as follows; **M** is decomposed for the suitable topology in the form

$$\mathbf{M} = \sum_{\alpha \in G} \mathbf{A}\, U_{\alpha},$$

where $\{U_\alpha\}_{\alpha \in G}$ is a unitary representation of G in **M** such that $U^*\mathbf{A}U \subset \mathbf{A}$. This class of the extension was inspired from the group extension theory and

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1) A W^*-algebra means a weakly closed self-adjoint operator algebra with the identity on a Hilbert space and a factor means a W^*-algebra whose center consists of scalar multiples of the identity.

2) By an isomorphism between W^*-algebras, we always understand a $*$-isomorphism, i.e. an automorphism of a factor means a $*$-automorphism.