TOPOLOGICAL REPRESENTATION OF C*-ALGEBRAS

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Let $\boldsymbol{A}$ be a C^*-algebra, Ω the structure space of $\boldsymbol{A}$, i.e. the space of all primitive ideals in $\boldsymbol{A}$ with hull-kernel topology. At every point P of Ω we associate a primitive C^*-algebra $\boldsymbol{A}/P$ (which we denote by $\boldsymbol{A}(P)$) and we may associate for any element $a \in \boldsymbol{A}$ the function $a(P)$ whose value at P is the homomorphic image of a in $\boldsymbol{A}(P)$. Then the most difficult parts of the non-commutative structure theory of C^*-algebras are the restrictions such as to destroy the main feature of the commutative case——the Gelfand representation of $\boldsymbol{A}$ by the *continuous* function $a(P)$ on Ω. Even if Ω is a Hausdorff space, it has long been observed hopeless to discuss the continuity of the function $a(P)$ since Kaplansky [7] proposed a method to study the structure of general C^*-algebras and instead of these discussions the continuity of the function $\|a(P)\|$ was studied. Unfortunately this property does not give directly the suitable topological representation of algebras.

On the other hand, in [11], in the case that $\boldsymbol{A}$ satisfies the condition that any irreducible representation of $\boldsymbol{A}$ is n-dimensional (such a C^*-algebra is called n-dimensionally homogeneous) we have defined a topology in the set $\mathscr{B} = \bigcup_{P \in \Omega} \boldsymbol{A}(P)$ and represented $\boldsymbol{A}$ as the algebra of all $\mathscr{B}$-valued functions $a(P)$ on Ω with $a(P) \in \boldsymbol{A}(P)$ which is continuous in this topology (we call these functions the (continuous) cross-sections of $\mathscr{B}$).

Now the above treatment offers a non-commutative model of the classical Gelfand representation theorem in the case that the structure space Ω is a Hausdorff space. Is it always possible to define a natural topology in the set $\mathscr{B} = \bigcup_{P \in \Omega} \boldsymbol{A}(P)$ so that $\boldsymbol{A}$ is represented as the algebra of all continuous cross-sections of $\mathscr{B}$ vanishing at infinity ? It is the main purpose of this paper to give a positive answer for this question and to analyse the algebras by their topological representations.

§1 and §2 are devoted to define a suitable topology in $\mathscr{B}$ in somewhat general situations and to discuss the general structure theory of algebras of cross-sections. Some fundamental results corresponding to the algebras of continuous functions are proved here, including the Stone-Weierstrass theorem and as a direct consequence of their results we can settle the problems remained unsolved in Kaplansky [7].

In §3 we treat the above mentioned problem stating our result in rather