ON CONTACT STRUCTURE OF HYPERSURFACES IN COMPLEX MANIFOLDS, I

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J.W.Gray [3], W. M. Boothby and H.C.Wang [1] introduced the notion of contact and almost contact structures and investigated it from the global viewpoint. An almost contact structure is one of an odd-dimensional manifold such that the structural group of its tangent bundle is reducible to the product of a unitary group with the one-dimensional identity group. It is comparable to almost complex structure of even-dimensional manifolds. S.Sasaki and Y.Hatakeyama [8, 9] proved that an almost contact structure can be represented as a totality of a tensor field and two vector fields satisfying certain conditions. It enables us to research properties of almost contact structures by use of tensor calculus.

In this paper we shall always assume that treated hypersurfaces are orientable. We shall show that a hypersurface in an almost complex manifold has an almost contact structure and that a hypersurface in an almost Hermitian manifold has an almost contact metric structure. Next we shall seek for a condition in order that a hypersurface in a Kählerian manifold has a contact structure. As a consequence we shall be able to obtain an extensive class of contact manifolds, which includes odd-dimensional spheres known as the simplest examples of contact manifolds. Finally we shall investigate the converse problem of imbedding of an almost contact or contact manifolds into an almost complex or complex manifold.

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1. Almost complex structure and almost contact structure. Let M be a $2p$-dimensional differentiable manifold covered with local coordinate systems (x^κ)[1]. An almost complex structure in M is by definition a (1, 1)-tensor field $F = (F_\lambda^{\ \kappa})$ satisfying the equation

$$FF = -E : F_\mu^{\ \lambda} F_\lambda^{\ \kappa} = -\delta_\mu^\kappa, \tag{1. 1}$$

where $E = (\delta_\mu^\kappa)$ is the unit tensor field in M. A manifold M with such a structure F is called an almost complex manifold. Improving the operators of J.A.Schouten and K.Yano [10], M. Obata [6] defined the following operators,

1) In this paper, Geeek indices run on the range $1, \cdots, 2p$, and small Latin indices on the range $1, \cdots, 2p-1$. Capital Latin indices run on the range $1, \cdots, 2p-1$ of small ones and an additional symbol ∞.