ON ABSOLUTE RIESZ SUMMABILITY FACTORS

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1. In this note sufficient conditions are given for a series $\sum a_n\epsilon_n$ to be summable $|R, \lambda, \kappa|$ whenever $\sum a_n$ is bounded (R, λ, κ). We shall restrict ourselves to integer values of κ. The non-integer case appears to present considerable difficulties—it is hoped to deal with it in a further note. When $\lambda_n = n$, our result is an alternative version of a theorem on absolute Cesàro summability factors (see Pati and Ahmad [7]); the equivalence of (R, n, κ), (C, κ) and $|R, n, \kappa|$, $|C, \kappa|$ summability being well known (Hobson [3], 90-98; Hyslop [4]).

In Section 4 we shall prove the following

THEOREM. *Suppose that the sequence of positive numbers* $\{\lambda_n\}$ *increases to infinity, and that*

(*a*) $0 < a \leqq \Delta\lambda_n/\Delta\lambda_{n-1} \leqq A$, *a,A constants. If* $A^\kappa(\omega) = O(\omega^\kappa)$, $\kappa = 0,1,2,\cdots$, *where* $A^\kappa(\omega)$ *is defined in* §2, *and*

(i) $\sum |\epsilon_n| < \infty$,

(ii) *there exists a function* $g(u)$, *defined for* $u \geqq \lambda_0$, *and a number* α, *such that for* $\nu = 0,1,\cdots$,

$$\epsilon_\nu = \alpha + \int_{\lambda_\nu}^{\infty} (u - \lambda_\nu)^\kappa \, dg(u) \qquad \textit{with} \int_{\lambda_0}^{\infty} u^\kappa |dg(u)| < \infty,$$

then $\sum a_n\epsilon_n$ *is summable* $|R, \lambda, \kappa|$.

Although we shall be concerned only with the sufficiency of conditions (i) and (ii), we remark that (ii) is necessary without any restriction on λ_n. For it has been shown that (ii) is necessary for $\sum a_n\epsilon_n$ to be summable (R, λ, μ) whenever $\sum a_n$ is summable (R, λ, κ), $\mu \geqq 0$, $\kappa \geqq 0$ (see Maddox [5]). Since $|R, \lambda, \mu|$ implies (R, λ, μ) summability, the result follows. We note also that $\alpha = 0$. For by (i) $\epsilon_n = o(1)$, and by (ii) $\epsilon_n = \alpha + o(1)$. Some remarks have been made on the condition (a) by Maddox [5]. The necessity of (i) has been