ON A FREE RESOLUTION OF A DIHEDRAL GROUP

SUGURU HAMADA

(Received December 20, 1962)

A free resolution of a group G is an exact sequence

$$\cdots \xrightarrow{d_{i+1}} X_i \xrightarrow{d_i} X_{i-1} \xrightarrow{d_{i-1}} \cdots \longrightarrow X_1 \xrightarrow{d_1} X_0 \xrightarrow{\varepsilon} Z \longrightarrow 0,$$

where X_i $(i=0,1,2,\cdots)$ are free G-modules, d_i,ε are G-homomorphisms, and Z is the ring of rational integers, on which G operates trivially.

For a cyclic group, we have the well-known simple free resolution. S. Takahashi [2] constructed a free resolution of abelian groups, and applied it to local number field theory, etc.

In this note, we construct a free resolution of a dihedral group, and decide n-dimensional cohomology groups for some modules. The author is grateful to Prof. T. Tannaka and Prof. H. Kuniyoshi who gave him this theme, encouragement and many suggestions.

1. Let G be a dihedral group, i.e. a group generated by s and t with relations $s^{2^l} = 1$, $t^2 = 1$, and $tst = s^{2^l - 1}$, where $l \geqq 2$.

We introduce the notations:

$$\Delta_1 = 1 - s, \qquad \Delta_2 = 1 - t, \qquad \Delta_3 = 1 - st,$$
$$N_1 = 1 + s + \cdots + s^{2^l - 1},\ N_2 = 1 + t,\ N_3 = 1 + st,$$

$\Lambda_0 = Z[s]$ group ring of the subgroup generated by s over Z,

$\Lambda = Z[G]$ group ring of G over Z.

Then, it follows $\Lambda = \Lambda_0 + \Lambda_0 t$ (direct), $N_3\Delta_1 = \Delta_1\Delta_2$, $\Delta_3\Delta_1 = \Delta_1 N_2$, $N_1\Delta_3 = \Delta_2 N_1$, and $N_1 N_3 = N_2 N_1$.

LEMMA 1. *We consider the following equations in* Λ

(1) $\qquad XN_i = 0 \qquad (i = 1,2,3)$

(2) $\qquad Y\Delta_i = 0 \qquad (i = 1,2,3)$

(3) $\qquad X\Delta_1 + Y\Delta_2 = 0$

and

(4) $$\begin{cases} XN_1 + YN_3 = 0 \\ Y(-\Delta_1) + WN_2 = 0. \end{cases}$$