ON NORMAL ALMOST CONTACT STRUCTURES WITH A REGULARITY

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Introduction. This paper is a continuation of the previous paper [4] in which we have proved, among others, that the bundle space of a principal circle bundle over a complex manifold, which has a connection satisfying certain conditions, admits a normal almost contact structure (cf. Theorem 6 [4]). In this paper we first consider the converse of the above theorem, and we shall call such a bundle, for the sake of simplicity, a contact bundle over a complex manifold (§1. Theorem 1).

In §2 we consider the period function of a regular closed vector field (Def. 3) and we prove Theorem 4 which says that the period function of a regular closed analytic vector field X on a complex manifold M is the real part of a holomorphic function on M if JX is also a closed vector field on M, J being the complex structure tensor of M. Using this theorem we shall prove that if the vector field ξ of a normal almost contact structure (ϕ, ξ, η) is a regular closed vector field, the period function of ξ is necessarily constant. From this we shall see that there is no other example of normal almost contact structures than the examples constructed in Theorem 6 [4], at least, when the vector field ξ is a closed vector field.

In §3 we consider the family of contact bundles over a complex manifold M_0 and we shall finally show that two contact bundles are isomorphic if and only if there exists a diffeomorphism f_0 of M_0 onto itself such that $f_0^*\overline{\Omega} = \Omega$, where Ω and $\overline{\Omega}$ are associated 2-forms on M_0 to each contact bundle, when M_0 is simply connected (cf. Def. 1).

1. Contact bundles over complex manifolds. Let $M(M_0, S^1, \pi)$ be a principal circle bundle over a (always C^∞-) differentiable manifold M_0, S^1 being the 1-dimensional torus and π being the projection of M onto M_0. Let $\Sigma = (\phi, \xi, \eta)$ be a normal almost contact structure (cf. Def. 2 [4]) on M. The Lie algebra $\mathfrak{r}$ of S^1 being identified with the real number field R, we shall now suppose that η is a connection form on M and that ξ is a vertical fundamental vector field A^* corresponding to the unit vector A of $\mathfrak{r}$. As in [4] we shall denote by $\mathfrak{V}(M)$ the Lie algebra of vector fields on M.

In the sequel we shall often denote the differential of a differentiable map f by the same letter f. We shall now prove the following theorem[1)] which

1) Y. Hatakeyama obtained similar results in Tôhoku Math. Journ., 15(1963), pp. 176–181.