A VERSION OF THE CENTRAL LIMIT THEOREM FOR TRIGONOMETRIC SERIES

SHIGERU TAKAHASHI

(Received August 2, 1964)

1. The central limit problem in the theory of probability is to determine the conditions under which the distribution functions of sums of random variables should converge [2]. In the case of independent variables the problem was solved completely. The studies on this problem for dependent variables are separated into two ways. One is to specify the dependency of variables by their conditional probabilities and one of the best known result is due to S. Bernstein [1]. The other is to specify the functional form of the variables and in this direction R. Salem and A. Zygmund proved the central limit theorem for lacunary trigonometric series [3].

THEOREM OF SALEM AND ZYGMUND. *Let*

$$L_N(t) = \sum_{k=1}^{N} c_k \cos 2\pi m_k(t+\phi_k)\,, \quad m_{k+1}/m_k \geqq q > 1\,,$$

where $\{c_k\}$ *and* $\{\phi_k\}$ *be arbitrary sequences of real numbers for which*

$$\| L_N \| = \left(\frac{1}{2}\sum_{k=1}^{N} c_k^2\right)^{1/2} \to +\infty \quad \textit{and} \quad c_N = o(\| L_N \|)\,, \quad \textit{as} \quad N \to +\infty\,.$$

Then we have, for any set $E \subset [0,1]$ *of positive measure and any real number* ω,

$$\lim_{N\to\infty} \frac{1}{|E|}\, |\{t\,;\ t\in E,\ L_N(t)/\| L_N \| \leqq \omega\}| = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\omega} \exp\left(-\frac{u^2}{2}\right) du.^{*)}$$

This theorem shows that the asymptotic behavior of a lacunary trigonometric series resembles that of independent random variables.

The purpose of the present note is to prove a version of the central limit theorem for trigonometric series not necessarily lacunary. Throughout this

*) For a measurable set E, $|E|$ denotes its Lebesgue measure.