SATURATION OF LOCAL APPROXIMATION BY LINEAR POSITIVE OPREATORS

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1. Introduction and inverse theorem. Let $f(x)$ be an integrable function, with period 2π and let its Fourier series be

$$(1)\qquad S[f] \equiv \sum_{k=0}^{\infty} A_k(x) \equiv \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) .$$

If the positivity of $f(x)$ implies the positivity of a linear operator $L_n(f, x)$, the operator is called a linear positive operator.

Let $\rho_k^{(n)}$ $(k=0, 1, 2, \cdots, \rho_0^{(n)}=1)$ be the "summating" function and consider a family of linear positive operators

$$(2)\qquad L_n(f, x) = \sum_{k=0}^{\infty} \rho_k^{(n)} A_k(x) .$$

Let us suppose that for a positive constant C, we have

$$(3)\qquad \lim_{n\to\infty} \frac{1-\rho_k^{(k)}}{1-\rho_1^{(n)}} = Ck^2 \quad (k = 1, 2, \cdots) .$$

The purpose of the present paper lies in considering local saturation by linear positive operators. Throughout the paper the norms should be taken with respect to the variable x and the subscript p $(1 \leqq p \leqq \infty)$ to L^p-norm will be generally omitted. Another convention is that the space (C) is meant by the notation L^∞, and the interval $[a, b]$ is an arbitrary subinterval of $[0, 2\pi]$. Thus the class $\mathrm{Lip}(\alpha, p)$ with $p = \infty$ reduces to $\mathrm{Lip}\,\alpha$. Also, let us write

$$\| L_n(f, x) - f(x) \|_{(a,b)} \equiv \left(\int_a^b | L_n(f, x) - f(x)|^p \, dx \right)^{\frac{1}{p}}$$

and

$$\mathrm{Lip}\,(1, p;\, a, b) \equiv \{f(x) | \sup_{|h|\leqq\delta} \| f(x+h) - f(x)\|_{(a,b)} = O(\delta)\} .$$