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ASYMPTOTIC BEHAVIORS IN FUNCTIONAL DIFFERENTIAL EQUATIONS.*)

JUNJI KATO

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Introduction. The asymptotic equivalence between a linear system and its perturbed system has been discussed by many authors (for references, see [10]). Two systems

$$\dot{x} = X(t, x) \tag{1}$$

and

$$\dot{x} = Y(t, x) \tag{2}$$

are said to be asymptotically equivalent, if the following condition is satisfied: For any solution $x(t; x_0, t_0)$ of one of the systems (1) and (2), we can find a solution of the other system, which tends to $x(t; x_0, t_0)$ as $t \to \infty$. However, for example, the systems

$$\dot{x} = 0$$

and

$$\dot{x} = x^2 e^{-t}$$

are not asymptotically equivalent in the sense above. Clearly, in the condition above, if we take t_0 suitably large according to the norm of x_0, then we can have the same conclusion. In the case, we shall say that they are eventually asymptotically equivalent.

In the previous papers, we have discussed the eventually asymptotic equivalence between more general systems and their perturbed systems, under the assumption that perturbation terms satisfy a special type of Lipschitz conditions [8, 9] or some type of integrabilities [10].

In this article, under much weaker condition, we shall discuss the eventually asymptotic equivalence between systems of functional differential equations and

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