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ON A CLASS OF OPERATORS

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1. We consider bounded linear operators on a Hilbert space H. Denote by $\sigma(T)$ the spectrum, by $\sigma_p(T)$ the point spectrum and by $\pi(T)$ the approximate point spectrum of an operator T. As in [3], an operator T is said to be of class (N) in case $\|T^2x\| \geqq \|Tx\|^2$ for all unit vectors $x \in H$. A. Wintner [8] calls an operator T normaloid if $\|T\| = \sup\{|(Tx, x)| : x \in H, \|x\|=1\}$. It is known that T is normaloid if and only if $\|T\| = \sup\{|\lambda| : \lambda \in \sigma(T)\}$ or equivalently, $\|T^n\| = \|T\|^n$ for $n = 1, 2, \cdots$. If T is a hyponormal operator, that is $\|Tx\| \geqq \|T^*x\|$ for all $x \in H$, then T is of class (N). In fact, if T is a hyponormal operator, we have

$$\|Tx\|^2 = (T^*Tx, x) \leqq \|T^*(Tx)\| \leqq \|T^2x\|,$$

for any unit vector $x \in H$.

2. In this section we prove some theorems on an operator of class (N). The following theorem is suggested by [6] and [7].

THEOREM 1. *For an operator T of class (N),*

(i) *T is normaloid,*

and

(ii) *T^{-1} is also of class (N) if T is invertible.*

PROOF. To prove (i), it is sufficient to show that $\|T^nx\| \geqq \|Tx\|^n$ for each unit vector $x \in H$ and $n=1, 2, \cdots$. If $n \leqq 2$, the inequality is obvious by the definition of class (N). Suppose that $\|T^kx\| \geqq \|Tx\|^k$ for $k=1, 2, \cdots, n$ and $x \in H$, $\|x\|=1$. Then

$$\|T^{n+1}x\| = \|Tx\| \left\| T^n \frac{Tx}{\|Tx\|} \right\| \geqq \|Tx\| \left\| T \frac{Tx}{\|Tx\|} \right\|^n$$

$$= \|Tx\|^{1-n} \|T^2x\|^n \geqq \|Tx\|^{1-n} \|Tx\|^{2n} = \|Tx\|^{n+1}$$