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ON BOREL-TYPE METHODS, II

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1. Introduction. In this paper, we define, and investigate the properties of the strong Borel-type methods $[B', \alpha, \beta]_p$, $[B, \alpha, \beta]_p$, which, when the index $p=1$, reduce to the methods $[B', \alpha, \beta]$, $[B, \alpha, \beta]$ considered in [1]. We use * to designate generalization of theorems, lemmas and definitions of [1]: e.g. Theorem 3* is a generalization of Theorem 3 of [1].

Suppose that σ, a_n $(n = 0, 1, \cdots)$ are arbitrary complex numbers, that $\alpha > 0$, that β is real and that N is a positive integer greater than $-\beta/\alpha$. Whenever $q > 1$, q' denotes the number conjugate to q, so that

$$\frac{1}{q} + \frac{1}{q'} = 1 .$$

Let x be a real variable in the range $[0, \infty)$: in all limits and order relations involving x, it is to be understood that $x \to \infty$.

Let

$$s_n = \sum_{\nu=0}^{n} a_\nu , \quad s_{-1} = 0 , \quad \sigma_N = \sigma - s_{N-1} ,$$

and define Borel-type sums

$$a_{\alpha,\beta}(x) = \sum_{n=N}^{\infty} \frac{a_n x^{\alpha n+\beta-1}}{\Gamma(\alpha n+\beta)} ; \qquad s_{\alpha,\beta}(x) = \sum_{n=N}^{\infty} \frac{s_n x^{\alpha n+\beta-1}}{\Gamma(\alpha n+\beta)} .$$

It is known that the convergence of either series for all $x \geqq 0$ implies the convergence, for all $x \geqq 0$, of the other.

Borel-type means are defined by

$$A_{\alpha,\beta}(x) = \int_0^x e^{-t} a_{\alpha,\beta}(t)\, dt ; \qquad S_{\alpha,\beta}(x) = \alpha e^{-x} s_{\alpha,\beta}(x) .$$