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SIMPLY INVARIANT SUBSPACES

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Our subject is a theorem on simply invariant subspaces of $L^p_{\mathfrak{h}}$, the usual L^p-space taking values in a Hilbert space $\mathfrak{h}$. Let X be a compact Hausdorff space and A a Dirichlet algebra on X. We shall fix a non-negative finite Borel measure m on X such that

$$f \longrightarrow \int f dm \qquad\qquad (f \in A)$$

defines a multiplicative linear functional on A. Define A_0 to be the set

$$A_0 = \{f \in A; \int f dm = 0\} .$$

Let $\mathfrak{h}$ be a separable Hilbert space and let $L^p_{\mathfrak{h}}$ $(1 \leqq p \leqq \infty)$ denote the space of $\mathfrak{h}$-valued functions on X which are weakly measurable and whose norms are in scalar $L^p(dm)$. $L^2_{\mathfrak{h}}$ is a Hilbert space for the inner product

$$(f, g) = \int (f(x), g(x))_{\mathfrak{h}} dm$$

where the inner product on the right is the one in $\mathfrak{h}$. We define $A_{\mathfrak{h}}$ by $A \otimes_\lambda \mathfrak{h}$, the completion of the algebraic tensor product $A \otimes \mathfrak{h}$ under the uniform norm in $C(X, \mathfrak{h})$ (the space of all $\mathfrak{h}$-valued continuous functions on X). For $1 \leqq p < \infty$ we define $H^p_{\mathfrak{h}}$ by

$$H^p_{\mathfrak{h}} = [A_{\mathfrak{h}}]_p$$

the closure of $A_{\mathfrak{h}}$ in $L^p_{\mathfrak{h}}$ and we define $H^\infty_{\mathfrak{h}}$ by

$$H^\infty_{\mathfrak{h}} = H^1_{\mathfrak{h}} \cap L^\infty_{\mathfrak{h}} .$$