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SOME CRITERIA FOR THE ABSOLUTE SUMMABILITY OF A FOURIER SERIES

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Let

$$f(x) \sim a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) = \sum_{n=0}^{\infty} A_n(x) \tag{1}$$

and $\sigma_n^k(x) (k > -1)$ denote the n-th (C, k) mean of Fourier series (1). If the series

$$\sum_{n=0}^{\infty} |\sigma_n^k(x) - \sigma_{n-1}^k(x)|$$

is convergent, we say that the series (1) is absolutely summable (C, k) or summable $|C, k|$. We denote the integral modulus of continuity of f by

$$\omega_p(t, f) = \sup_{0<h<t} \left\{ \int_{-\pi}^{\pi} |f(x+h) - f(x-h)|^p dx \right\}^{1/p} \quad (1 \leqq p < \infty).$$

H. C. Chow [1] proved the following theorem.

THEOREM A. *Let* $1 \leqq p \leqq 2$. *If*

$$\omega_p(t, f) = O\left\{ \left(\log \frac{1}{t} \right)^{-1-\delta} \right\} \qquad (\delta > 0),$$

then the series (1) *is summable* $|C, \alpha|$ *almost everywhere for* $\alpha > 1/p$.

On the other hand, P. L. Ul'yanov [5] proved the following theorem.

THEOREM B. *If*

$$\omega_2(t, f) = O\left\{ \left(\log \frac{1}{t} \right)^{-1/2-\delta} \right\} \qquad (\delta > 0),$$