Tôhoku Math. Journ.
20(1968), 333-345.

# ON CONTRAVARIANT $C$-ANALYTIC 1-FORMS IN A COMPACT SASAKIAN SPACE

YÔSUKE OGAWA

(Received February 5, 1968)

**Introduction**. Let $F_i{}^j$ be the complex structure tensor in a Kaehlerian space. If a 1-form $u$ leaves invariant the structure tensor $F_i{}^j$, then it is called a contravariant analytic 1-form. It is defined by the relation

$$\nabla_i u_j - F_i{}^a F_j{}^b \nabla_a u_b = 0 .$$

In a compact Kaehler Einstein space, we take a contravariant analytic 1-form $u$. Then there exist Killing 1-forms $v$ and $w$ such that $u$ can be written as

$$u_i = v_i + F_i{}^j w_j ,$$

which is known as a theorem of Matsushima [3]. We consider the analogy of this theorem in a compact $C$-Einstein space. Denote the structure tensors of a Sasakian space by $(\varphi_\lambda{}^\mu, \eta_\lambda, g_{\lambda\mu})$. Then it is known that a 1-form $u$ which leaves invariant the tensor $\varphi_\lambda{}^\mu$ is Killing in a compact contact space (S. Tanno [2]). Therefore if we take a $\varphi_\lambda{}^\mu$-preserving 1-form instead of a contravariant analytic 1-form in a Kaehlerian case, the analogy of the theorem of Matsushima is trivial.

In the former paper [6], we introduced the notion of $C$-Killing 1-forms on a Sasakian space. We call a 1-form $u$ $C$-Killing if it satisfies $\delta u=0$ and leaves invariant $g_{\lambda\mu} - \eta_\lambda \eta_\mu$. Especially if a $C$-Killing form $u$ satisfies $u' = i(\eta)u$ $=$constant, it is called special $C$-Killing. In a compact Sasakian space, it is known that a 1-form $u$ is special $C$-Killing if and only if it satisfies

$$\nabla_\lambda u_\mu + \nabla_\mu u_\lambda = 2u^\rho(\varphi_{\rho\lambda}\eta_\mu + \varphi_{\rho\mu}\eta_\lambda) .$$

Now we study the analogy of the theorem of Matsushima taking the $C$-Killing forms for Killing forms.

In a Sasakian space, we define a contravariant $C$-analytic 1-form $u$ by the relation