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FUNCTIONS OF L^p-MULTIPLIERS

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1. **Introduction**. Let Γ be a locally compact non-compact abelian group and $B(\Gamma)$ be the space of all Fourier-Stieltjes transforms of bounded measures on the dual group G of Γ. Then it is known that a function Φ on the interval $[-1,1]$ is extended to an entire function if and only if $\Phi(f)\in B(\Gamma)$ for all f in $B(\Gamma)$ with the range contained in $[-1,1]$ (see, for example, [10: p.135]).

A function φ defined on Γ is called an L^p-multiplier if for every $f\in L^p(G)$ there exists a function g in $L^p(G)$ so that $\varphi\hat{f}=\hat{g}$, where $\hat{f}$ denotes the Fourier transform of f. The set of all L^p-multipliers will be written by $M_p(\Gamma)$ and the norm of $\varphi\in M_p(\Gamma)$ is defined by

$$\|\varphi\|_{M_p(\Gamma)}=\sup\{\|g\|_{L^p(G)}:\ \|f\|_{L^p(G)}=1\}.$$

If we define the product in $M_p(\Gamma)$ by the pointwise multiplication, it is a commutative Banach algebra with identity.

It is well-known that $M_1(\Gamma)$ coincides with $B(\Gamma)$ with the norm of measures and $M_2(\Gamma)=L^\infty(\Gamma)$ isometrically. If $1\leqq q\leqq p\leqq 2$, then $M_q(\Gamma)\subset M_p(\Gamma)$ and if $1/p+1/p'=1$, then $M_p(\Gamma)=M_{p'}(\Gamma)$ isometrically.

Our main theorem is the following:

THEOREM 1. *Let Γ be a locally compact non-compact abelian group. Assume $1\leqq p<2$ and Φ is a function on $[-1,1]$. Then $\Phi(\varphi)\in M_p(\Gamma)$ for all φ in $M_1(\Gamma)$ whose range is contained in $[-1,1]$, if and only if Φ is extended to an entire function.*

2. **Equivalence of multiplier transforms**. In this section we shall show the equivalence of multiplier transforms which will be needed later.

A measurable function φ on the real line $\boldsymbol{R}$ is said to be regulated if there exists an approximate identity u_ε not necessarily continuous such that

$$\lim_{\varepsilon\to 0}\varphi*u_\varepsilon(x)=\varphi(x)$$

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