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SASAKIAN MANIFOLD WITH PSEUDO-RIEMANNIAN METRIC

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Introduction. Sasakian manifold with Riemannian metric is defined by S. Sasaki [5]. In this paper, we want to define Sasakian manifold with pseudo-Riemannian metric, and discuss the classification of Sasakian manifolds.

In section 1, we define a Sasakian manifold (with pseudo-Riemannian metric). In section 2, we define the model spaces of Sasakian manifolds which are used in section 4 for the classification of Sasakian manifolds of constant ϕ-sectional curvatures. In section 3, we discuss D-homothetic deformation which is defined by S. Tanno [9], and prove some fundamental lemmas concerning completeness of the deformed metric. In section 5, we prove that a Sasakian manifold, satisfying $R(X,Y)\cdot R=0$ for all tangent vectors X and Y, is of constant curvature. In section 6, we discuss a Sasakian manifold M^{2n+1} which is properly and isometrically immersed in E_s^{2n+2}.

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1. Preliminaries. Manifolds and tensor fields are supposed to be of class C^∞.

Let $M=M^{2n+1}$ be a connected differentiable manifold, and let ϕ, ξ and η be tensor fields of type (1, 1), (1, 0) and (0, 1), respectively, on M.

DEFINITION. (ϕ, ξ, η) is called an *almost contact structure* on M, if the followings are satisfied:

$$\eta(\xi)=1\,,$$

$$\eta(\phi(X))=0\,,\qquad X\in\mathfrak{X}(M)\,,$$

$$\phi^2(X)=-X+\eta(X)\xi,\qquad X\in\mathfrak{X}(M)\,.$$

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