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ON THE CONVERGENCE OF NONLINEAR SEMI-GROUPS

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1. Introduction. Let X be a Banach space and let $\{T(\xi);\ \xi \geqq 0\}$ be a family of (nonlinear) operators from X into itself satisfying the following conditions:

(i) $T(0) = I$(the identity) and $T(\xi + \eta) = T(\xi)T(\eta)$ for $\xi, \eta \geqq 0$.

(ii) For each $x \in X$, $T(\xi)x$ is strongly continuous in $\xi \geqq 0$.

We call such a family $\{T(\xi);\ \xi \geqq 0\}$ simply a *nonlinear semi-group.* If there is a non-negative constant c such that

(iii) $\|T(\xi)x - T(\xi)y\| \leqq e^{c\xi} \|x-y\|$ for $x, y \in X$ and $\xi \geqq 0$,

then a nonlinear semi-group $\{T(\xi);\ \xi \geqq 0\}$ is said to be *of local type.* (In particular, if $c = 0$, it is called a *nonlinear contraction semi-group.*) We define the *infinitesimal generator* A_0 of a nonlinear semi-group $\{T(\xi);\ \xi \geqq 0\}$ by

$$A_0x = \lim_{\delta\to 0+} \delta^{-1}(T(\delta) - I)x \tag{1.1}$$

and the *weak infinitesimal generator* A' by

$$A'x = \text{w-}\lim_{\delta\to 0+} \delta^{-1}(T(\delta) - I)x, \tag{1.2}$$

if the right sides exist. (The notation "lim" ("w-lim") means the strong limit (the weak limit) in X.)

REMARK. In case of *linear* semi-groups, it is well known that the weak infinitesimal generator coincides with the infinitesimal generator.

H. F. Trotter [9] proved the following convergence theorem of *linear* semi-groups.

THEOREM. *Let* $\{T_n(\xi);\ \xi \geqq 0\}_{n=1,2,3,\cdots}$ *be a sequence of semi-groups (of linear operators) of class* (C_0) *satisfying the stability condition*

$$\|T_n(\xi)\| \leqq Me^{\omega\xi} \ \textit{for} \ \ \xi \geqq 0,\ n = 1, 2, 3, \cdots,$$

where M *and* ω *are independent of* n *and* ξ. *Let* A_n *be the infinitesimal generator of* $\{T_n(\xi);\ \xi \geqq 0\}$ *and define* $Ax = \lim_n A_nx$.