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KÄHLER SUBMANIFOLDS OF HOMOGENEOUS ALMOST HERMITIAN MANIFOLDS

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Let $M=(M, J, <\ ,\ >)$ be an almost Hermitian manifold, i.e., J is an almost complex structure on M, $<\ ,\ >$ is a Riemannian metric on M, and $<JX, JY> = <X, Y>$ for all vector fields X, Y on M. (Throughout this paper each almost Hermitian manifold M that we consider is always assumed to have a specific Riemannian metric and almost complex structure associated with it.) We define the *Kähler deficiency* of M to be the least integer $d(M)$ such that any Kähler submanifold of M must have (real) dimension $\leqq d(M)$. The purpose of this paper is to estimate the Kähler deficiency of certain homogeneous almost Hermitian manifolds of positive Euler characteristic.

The proofs of the following two propositions are easy.

PROPOSITION 1. *Let $M = M_1 \times \cdots \times M_r$ be a Riemannian product manifold. Suppose that $M, M_1, \cdots, M_r$ are all almost Hermitian with almost complex structures $J, J_1, \cdots, J_r$ such that $J = J_1 \oplus \cdots \oplus J_r$. Then*

$$d(M) = d(M_1) + \cdots + d(M_r)\,.$$

PROPOSITION 2. *Let M be an almost Hermitian manifold and suppose $\widetilde{M}$ is a covering space of M. Then M is almost Hermitian and $d(\widetilde{M}) = d(M)$.*

Now let M be a compact homogeneous space of positive Euler characteristic. Because of Proposition 2, we may assume that M is simply connected and that $M=G/K$ where G is a compact semisimple Lie group acting effectively on M and K is the isotropy group at some point $p \in M$. Then rank $G=$rank K. We assume that the metric of M is determined by a biinvariant metric on G.

Denote by θ an automorphism of G of order 3 and assume that K is the

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