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ON THE ABSOLUTE NÖRLUND SUMMABILITY OF THE FACTORED FOURIER SERIES

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1. Let $\{s_n\}$ denote the n-th partial sum of a given infinite series $\sum a_n$. Let $\{p_n\}$ be a sequence of constants, real or complex and let

$$P_n = p_0 + p_1 + \cdots\cdots\cdots\cdots + p_n,\ P_{-1} = p_{-1} = 0.$$

The sequence $\{t_n\}$, given by

$$t_n = \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k = \frac{1}{P_n}\sum_{k=0}^{n} P_k a_{n-k} \tag{1.1}$$

defines the Nörlund means of the sequence $\{s_n\}$ generated by the sequence $\{p_n\}$.

Then, the series $\sum a_n$ is said to be summable $|N, p_n|$, if the sequence $\{t_n\}$ is of bounded variation, that is, the series

$$\sum_n |t_n - t_{n-1}| \tag{1.2}$$

is convergent.

When the special cases in which $p_n = \dfrac{\Gamma(n+\alpha)}{\Gamma(\alpha)\Gamma(n+1)}$, $\alpha > 0$, and $p_n = \dfrac{1}{n+1}$, summability $|N,p_n|$ are the same as the summability $|C,\alpha|$ and the absolute harmonic summability, respectively.

2. Let $f(t)$ be a periodic function with period 2π and Lebesgue integrable over $(-\pi, \pi)$. We assume, without any loss of generality, that the constant term in the Fourier series of $f(t)$ is zero and that the Fourier series of $f(t)$ is given by

$$\sum_{n=1}^{\infty}(a_n\cos nt + b_n\sin nt) = \sum_{n=1}^{\infty} A_n(t). \tag{2.1}$$