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SASAKIAN HYPERSURFACES IN A SPACE OF CONSTANT CURVATURE

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1. Introduction. In the previous paper [10], we defined a Sasakian manifold with pseudo-Riemannian metric, and proved that a Sasakian manifold M^{2n+1} which is properly and isometrically immersed in a pseudo-Riemannian manifold $\widetilde{M}^{2n+2}$ of constant curvature zero is of constant curvature one. It is an extension of Corollary for Theorem 2 in Tashiro-Tachibana [13]. In this paper, we prove that a Sasakian manifold M^{2n+1} (with a pseudo-Riemannian metric) which is properly and isometrically immersed in a pseudo-Riemannian manifold $\widetilde{M}^{2n+2}$ of constant curvature $\tilde{c}\neq 1$ is of constant curvature 1 (Theorem 1). In the case when $\tilde{c}=1$, we need an additional condition; namely, the Sasakian manifold to be η-Einstein, then it is of constant curvature 1 (Theorem 4). Some related results on almost contact hypersurfaces are found in Tashiro [12], Kurita [3], Tashiro-Tachibana [13] and Okumura [6], [7].

In this paper, we call a Sasakian manifold with pseudo-Riemannian metric to be a pseudo-Sasakian manifold.

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2. Let $(\widetilde{M}^{2n+2}, \tilde{g})$ be a pseudo-Riemannian manifold of constant curvature $\tilde{c}$. Let $M^{2n+1}(\phi, \xi, \eta, g)$ be a pseudo-Sasakian manifold which is isometrically immersed in $(\widetilde{M}^{2n+2}, \tilde{g})$. Then we have the formulas of Gauss and Codazzi:

$$R(X, Y) = \tilde{c}X \wedge Y + \varepsilon AX \wedge AY\,, \tag{1}$$

$$(\nabla_X A)Y - (\nabla_Y A)X = 0\,, \tag{2}$$

where $X\wedge Y$ denotes an endomorphism $Z \rightarrow g(Y, Z)X - g(X, Z)Y$, A is the field of the second fundamental form operators which corresponds to the field of unit normal vectors ζ to M^{2n+1} and $\varepsilon = \tilde{g}(\zeta, \zeta)$, $\varepsilon = +1$ or -1 (L. P. Eisenhart [1]). From (1), we get

$$R(X, \xi)Y = \tilde{c}\{\eta(Y)X - g(X, Y)\xi\} + \varepsilon\{\eta(AY)AX - g(AX, Y)A\xi\}\,. \tag{3}$$