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A NON-COMMUTATIVE THEORY OF INTEGRATION FOR A SEMI-FINITE AW^*-ALGEBRA AND A PROBLEM OF FELDMAN

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1. Introduction. In [5], I.Kaplansky introduced a class of C^*-algebras called AW^*-algebras. For these, while being algebraically defined, much of the Murray-von Neumann structure theory for von Neumann algebras, in particular, the lattice structure theory of the set of projections can be developed. Dixmier showed that this class of AW^*-algebras is exactly broader than that of von Neumann algebras [1]. Therefore, it is an interesting problem for us to investigate the difference between AW^*-algebras and von Neumann algebras. From this point of view, we shall extend Feldman's result on "Embedding of AW^*-algebras" to semi-finite AW^*-algebras, that is, we shall show that a semi-finite AW^*-algebra with a separating set of states which are completely additive on projections (c. a. states) has a faithful representation as a semi-finite von Neumann algebra on some Hilbert space (Theorem 5. 2). He showed that a finite AW^*-algebra which possesses a separating set of c. a. states admits a faithful representation as a von Neumann algebra [3].

In the previous paper [7], we constructed the algebra $\mathcal{C}$ of "measurable operators" for a semi-finite AW^*-algebra M in algebraic fashion and studied the structure of $\mathcal{C}$. Throughout this paper, we always assume M to be a semi-finite AW^*-algebra with a separating set $\mathfrak{S}$ of c. a. states and $\mathcal{C}$ to be the algebra of "measurable operators" for it.

The contents of this paper are as follows. Section 2 is preliminary. We review briefly the definitions and elementary properties of M which will be used later. In section 3, along the same lines with [10], we shall prove the existence theorem of a dimension function (Theorem 3. 2) for M and introduce the notion of convergence nearly everywhere of sequences in $\mathcal{C}$. Section 4 concerns with the existence of a faithful semi-finite numerical trace τ on M and the non-commutative integration theory with respect to τ. We shall show that the set $\mathfrak{H}_\tau$ of square τ-integrable elements in $\mathcal{C}$ is a Hilbert space under a suitable norm (Theorem 4. 7). Section 5 is the main part of this paper and is devoted to prove the theorem: M can be represented faithfully as a semi-finite von Neumann algebra (Theorem 5. 2). As a corollary, we give the alternative proof of Theorem 2 in [6], more precisely, an AW^*-algebra of type I whose center is a W^*-algebra admits a