Tôhoku Math Journ.
22 (1970), 619-634.

OSCILLATORY PROPERTY OF SOLUTIONS OF SECOND ORDER DIFFERENTIAL EQUATIONS

TARO YOSHIZAWA

(Received on July 2, 1970)

In this paper we shall discuss oscillatory property of solutions of second order differential equations by applying Liapunov's second method. Consider an equation

$$(1)\qquad (r(t)x')' + f(t, x, x') = 0 \quad \left(' = \frac{d}{dt}\right),$$

where $r(t) > 0$ is continuous on $I = [0, \infty)$ and $f(t, x, u)$ is defined and continuous on $I \times R \times R$, $R = (-\infty, \infty)$. To discuss oscillatory property of solutions of (1), we consider an equivalent system

$$(2)\qquad x' = \frac{y}{r(t)}, \quad y' = -f\left(t, x, \frac{y}{r(t)}\right).$$

A solution $x(t)$ of (1) which exists in the future is said to be oscillatory if for every $T > 0$ there exists a $t_0 > T$ such that $x(t_0) = 0$. Moreover, the equation (1) is said to be oscillatory if every solution of (1) which exists in the future is oscillatory.

THEOREM 1. *Assume that there exist two continuous functions* $V(t, x, y)$ *and* $W(t, x, y)$ *which are defined on* $t \geqq T$, $x > 0$, $|y| < \infty$ *and* $t \geqq T$, $x < 0$, $|y| < \infty$, *respectively, where* T *can be large, and assume that* $V(t, x, y)$ *and* $W(t, x, y)$ *satisfy the following conditions;*

(i) $V(t, x, y) \to \infty$ *uniformly for* $x > 0$ *and* $-\infty < y < \infty$ *as* $t \to \infty$, *and* $W(t, x, y) \to \infty$ *uniformly for* $x < 0$ *and* $-\infty < y < \infty$ *as* $t \to \infty$,

(ii) $\dot{V}_{(2)}(t, x(t), y(t)) \leqq 0$ *for all sufficiently large* t, *where* $\{x(t), y(t)\}$ *is a solution of* (2) *such that* $x(t) > 0$ *for all large* t *and*

$$\dot{V}_{(2)}(t, x(t), y(t)) = \overline{\lim_{h \to 0^+}} \frac{1}{h} \{V(t + h, x(t + h), y(t + h)) - V(t, x(t), y(t))\},$$