A CLASS OF ALMOST CONTACT RIEMANNIAN MANIFOLDS

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1. Introduction. Recently S. Tanno has classified connected almost contact Riemannian manifolds whose automorphism groups have the maximum dimension [9]. In his classification table the almost contact Riemannian manifolds are divided into three classes: (1) homogeneous normal contact Riemannian manifolds with constant ϕ-holomorphic sectional curvature if the sectional curvature for 2-planes which contain ξ, say $K(X, \xi), > 0$, (2) global Riemannian products of a line or a circle and a Kaehlerian manifold with constant holomorphic sectional curvature, if $K(X, \xi) = 0$ and (3) a warped product space $L \times_f CE^n$, if $K(X, \xi) < 0$. It is known that the manifold of the class (1) in the above statement is characterized by some tensor equations; it has a Sasakian structure.

The purpose of this paper is to characterize the warped product space $L \times_f CE^n$ by tensor equations (§ 2) and study their properties. From the definition by means of the tensor equations it is easily verified that the structure is normal, but not quasi-Sasakian (and is hence not Sasakian). In § 2, we define a structure closely related to the warped product which is studied by Bishop-O'Neill [1] and prove the local structure theorem. In § 3 we study some properties of the structure. § 4 is devoted to a study of η-Einstein manifolds. In the section 5 we show one of the main theorems in this paper. In the last section we study invariant submanifolds.

We follow here the notations and the terminology of the Volume 1 of Kobayashi-Nomizu [4].

2. Definition and examples. It is well-known that the structure tensors (ϕ, ξ, η, g) of the almost contact Riemannian maifold M satisfy

(2.1) $$\phi\xi = 0 , \qquad \eta(\phi X) = 0 , \qquad \eta(\xi) = 1 ,$$

(2.2) $$\phi\phi X = -X + \eta(X)\xi , \qquad g(X, \xi) = \eta(X) ,$$

(2.3) $$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) ,$$

for any vector fields X and Y on M. It is known that the (ϕ, ξ, η, g)-structure is normal if and only if