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IMAGINARY ABELIAN NUMBER FIELDS WITH CLASS NUMBER ONE

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We have shown that there exist only a finite number of imaginary abelian number fields with relative class number $h_1 = 1$ [12 or 13]. There an upper bound of the conductors of such fields could be effectively determined, except for biquadratic fields of type (2, 2). Now Baker's and Stark's papers [3 and 10] show that an upper bound can be effectively determined also for those fields, because biquadratic fields of type (2, 2) with $h_1 = 1$ are generated by imaginary quadratic fields with $h_1 = 1$ or 2. So it is a problem of finite amount of calculation to determine all the imaginary abelian number fields with $h_1 = 1$. But an upper bound we can now obtain is too large to solve this problem explicitly. In this paper, we restrict ourselves to the class number (not the relative class number) one problem, and we give some remarks and upper bounds for some cases.

1. In this section we give some remarks which will be useful for the class number one problem. They are not essentially new results, but it will be convenient to remark here.

We define a field of type I to be an imaginary abelian number field which is generated by subfields of prime power conductors. When we write as $K = K_1K_2 \cdots K_r$ for a field of type I, we always mean that K_i are subfields of prime power conductors which are relatively prime. First proposition which is a corollary of genus theory shows that an imaginary abelian number field with class number one is of type I.

PROPOSITION 1. *Let K be an abelian number field of finite degree. Let $k = p_1^{e_1}p_2^{e_2} \cdots p_r^{e_r}$ be its conductor. If K has strict class number one, K is generated by subfields K_i whose conductors are $p_i^{e_i}$. Every K_i also has strict class number one.*

PROOF. K is contained in the field L of the k-th roots of unity. Let E_1 be the field of the $p_1^{e_1}$-th roots of unity, and let E_2 be the field of the $p_2^{e_2} \cdots p_r^{e_r}$-th roots of unity. Let T be the inertia subfield of K with respect to p_1. Then it holds $T = K \cap E_2$, and the Galois group of