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TRANSFORMATION OF THE GENERALIZED WIENER MEASURE UNDER A CLASS OF LINEAR TRANSFORMATIONS

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1. Introduction. Let C_w be the Wiener space consisting of continuous real valued functions $x(t)$ on $[0, 1]$ with $x(0) = 0$. It is the purpose of this paper to investigate the transformation of the generalized Wiener measure on C_w corresponding to the generalized Brownian motion process (i.e. Brownian motion process with nonstationary increments) when the elements of C_w are transformed by a Volterra integral equation of the second kind.

For $0 = t_0 < t_1 < \cdots < t_n \leqq 1$, let $\mathfrak{F}_{t_1 \cdots t_n}$ be the σ-field of subsets of C_w of the type

$$(1.1) \qquad E = \{x \in C_w; [x(t_1), \cdots, x(t_n)] \in B\}, \qquad B \in \mathfrak{B}^n$$

where $\mathfrak{B}^n$ is the σ-field of Borel sets in the n-dimensional Euclidean space R^n. Let $b(t)$ be a strictly increasing continuous function on $[0, 1]$. It is well known that if we define a set function m on $\mathfrak{F}_{t_1 \cdots t_n}$ by

$$(1.2) \qquad m(E) = \frac{1}{\left\{(2\pi)^n \prod_{i=1}^{n} [b(t_i) - b(t_{i-1})]^2\right\}^{1/2}} \int_{-\infty}^{\infty} (n) \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2} \sum_{i=1}^{n} \frac{(\xi_i - \xi_{i-1})^2}{b(t_i) - b(t_{i-1})}\right\} d\xi_1 \cdots d\xi_n$$

with $\xi_0 \equiv 0$, then m is well defined on the σ-field $\mathfrak{F}$ generated by the field $\mathfrak{F}_0$ which is the union of all the σ-fields $\mathfrak{F}_{t_1 \cdots t_n}$ and is in fact a probability measure on $(C_w, \mathfrak{F})$. (See for instance K. Itô [4] and P. Lévy [6].) Let $\mathfrak{F}^*$ be the Carathéodory extension of $\mathfrak{F}_0$ relative to m. Then $(C_w, \mathfrak{F}^*, m)$ is a complete probability measure space. We shall refer to $\mathfrak{F}^*$-measurability as Wiener measurability, and to m as the generalized Wiener measure corresponding to b.

The real valued function $X(t, x) = x(t)$, $x \in C_w$, $t \in [0, 1]$ is then a stochastic process with independent increments on the probability space $(C_w, \mathfrak{F}^*, m)$. In fact $X(0, x) = 0$ for every $x \in C_w$, and the increment $X(t'', x) - X(t', x)$ is distributed according to $N(0, b(t'') - b(t'))$, i.e. the probability distribution Φ of the above increment is a normal distribution with density