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ON THE MEAN CURVATURE FOR HOLOMORPHIC $2p$-PLANE IN KÄHLERIAN SPACES

Dedicated to Professor Shigeo Sasaki on his 60th birthday

SHUN-ICHI TACHIBANA

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Introduction. Let M^n be an n dimensional Riemannian space, and denote by $\rho(X, Y)$ the sectional curvature of a 2-plane spanned by vectors X and Y. For a q-plane π at a point P, we take an orthonormal base $\{e_\lambda\}$ of the tangent space $T_P(M)$ such that $e_1, \cdots, e_q$ span π. Such a base is called an adapted base for π. The mean curvature $\rho(\pi)$ for π is defined by

$$\rho(\pi) = \frac{1}{q(n-q)} \sum_{a=q+1}^{n} \sum_{i=1}^{q} \rho(e_i, e_a)$$

and independent of the choice of adapted bases for π.

In a recent paper [3], we have proved the following

THEOREM. *In an $n\,(>2)$ dimensional Riemannian space M^n, if the mean curvature for q-plane is independent of the q-plane at each point, then*

(i) *M^n is an Einstein space, for $q = 1$ or $n - 1$,*

(ii) *M^n is of constant curvature, for $n - 1 > q > 1$ and $2q \neq n$,*

(iii) *M^n is conformally flat, for $n - 1 > q > 1$ and $2q = n$.*

The converse is also true.

The purpose of this paper is to prove an analogous theorem in Kählerian spaces taking holomorphic $2p$-plane in place of q-plane in the above theorem.

1. Preliminaries.

In [3], the following has been proved.

LEMMA A. *Let $A = (a_{ij})$ be an $m \times m$ symmetric matrix whose diagonal elements are all zero. If $1 < p < m - 1$ and A satisfies*

$$\sum_{h,k=1}^{p} a_{i_h i_k} = 0$$

for any $i_1 < \cdots < i_p$ taken from $\{1, \cdots, m\}$, then A is the zero matrix.

We shall generalize this lemma as follows:

LEMMA 1.1. *Let $B = (b_{ij})$ be an $m \times m$ symmetric matrix. If $1 <$*