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CONFORMALLY FLAT HYPERSURFACES IN A CONFORMALLY FLAT RIEMANNIAN MANIFOLD

SEIKI NISHIKAWA AND YOSHIAKI MAEDA*)

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Introduction. A Riemannian manifold is called *conformally flat* if it is locally conformally equivalent to a Euclidean space, i.e., if each point of the manifold has a neighborhood where there exists a conformal diffeomorphism onto an open subset in a Euclidean space. Well-known examples of such manifolds which are also hypersurfaces in a Euclidean $(n+1)$-space E^{n+1} are the following: a Euclidean n-space E^n, a Euclidean n-sphere S^n, a right circular cylinder $E^{n-1}\times S^1$ and a Riemannian product manifold $S^{n-1}\times E^1$. It will then be natural to ask: Is there any other conformally flat hypersurface in E^{n+1} which is not conformally diffeomorphic to any of the above examples? Generalizing this question, we can pose the following problem: Classify the conformally flat hypersurfaces in a conformally flat Riemannian manifold up to conformal equivalence. This problem is attractive in conformal geometry.

As a first step to the above problem, we shall classify up to isometry local structures of conformally flat hypersurfaces, especially in a Riemannian manifold of constant curvature, and it is the main purpose of this paper. In fact, our study goes as follows. After preparing some basic definitions and formulas in §1, we shall determine, in §2, the types of the second fundamental forms of these hypersurfaces. The result is that *a hypersurface in a conformally flat Riemannian $(n+1)$-manifold $(n>3)$ is conformally flat if and only if at each point, at least $(n-1)$ eigenvalues of the second fundamental form are identical* (Theorem 3). Making use of this fact, in §3, we shall classify local structures of conformally flat hypersurfaces in a Riemannian manifold of constant curvature. The result is summarized as follows: *Let $M^n (n>3)$ be a conformally flat hypersurface of a Riemannian $(n+1)$-manifold of constant curvature. Then there exist four types (for details, see* §3*) of the local structures of M^n, all of which are loci of a moving $(n-1)$-submanifold $M^{n-1}(v)$ which is of constant curvature for each value of a parameter v* (Theorem 4).

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